SENIOR SECONDARY IMPROVEMENT PROGRAMME 2013

GRADE 12

MATHEMATICS

TEACHER NOTES

The SSIP is supported by

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TEACHER NOTES

SESSION 16.1

TOPIC: DATA HANDLING

Teacher Note: Data Handling makes up approximately 20% of Paper 2. Based on the analysis of the November 2011 exam, this is a section that learners generally perform well in. However, there are still key concepts that need attention. The following should be emphasised during this session: language (terminology used in data questions), use of the STAT mode on the calculator and how to interpret results and communicate conclusions. This session is designed in particular to help learners understand how to apply what they have learnt in grade 11 to answer questions regarding best fit and distribution of data. It is important that learners understand that it is crucial that they are able to interpret a set of data and communicate that.

LESSON OVERVIEW

Teacher Note: Remind learners of the concepts that they need to know from grade 11. The following concepts need to be understood: mean, mode, quartiles, range, interquartile range, five number summary, ogives, Box and Whisker plots, variance and standard deviation.

It is also important to remind learners of the formulae given on the formula sheet. Learners must familiarise themselves with that sheet.
Provide learners with tips regarding neatness, layout, notation and mark allocations.

1. Introduction session: 5 min
2. Typical exam questions:
   - Question 1: 15 min
   - Question 2: 10 min
   - Question 3: 10 min
   - Question 4: 5 min
3. Discussion: 5 min

SECTION A: TYPICAL EXAM QUESTIONS

QUESTION 1

The ages of the final 23 players selected by coach Carlos Perreira to play for Bafana Bafana in the 2010 FIFA World Cup are provided on the following page.

Position	Player	Age
1	Shu-Aib Walters	28
2	Siboniso Gaxa	26
3	Tshepo Masilela	25
4	Aaron Moekoena	29
5	Lucas Thwala	28
6	Macbeth Sibaya	32
7	Lance Davids	25
8	Siphiwe Tshabalala	25
9	Katlego Mphela	25
10	Steven Pienaar	28
11	Teko Modise	27
12	Reneilwe Letsholonyane	28
13	Kagisho Dikgacoi	25
14	Matthew Booth	33
15	Bernard Parker	24
16	Itumeleng Khune	22
17	Surprise Moriri	30
18	Siyabonga Nomvethe	32
19	Anele Ngcongca	22
20	Bongani Khumalo	23
21	Siyabonga Sangweni	28
22	Moeneeb Josephs	30
23	Thanduyise Khuboni	24

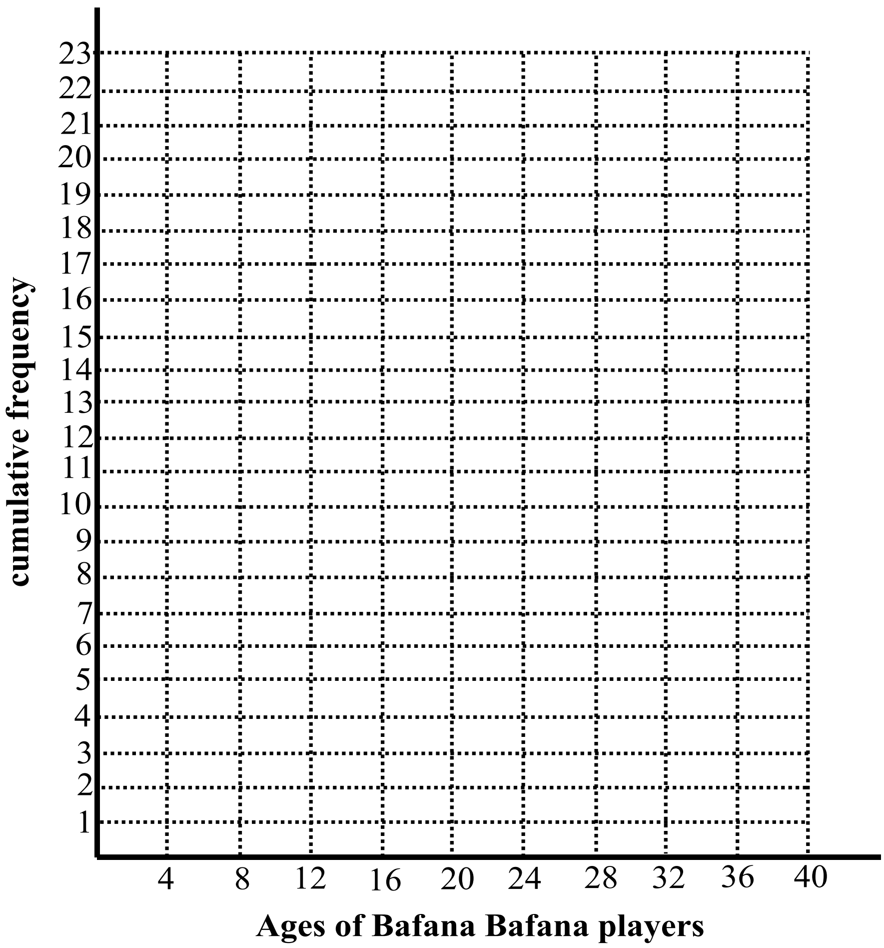
Source: www.2010 Fifa World Cup:final squads – MediaClubSouthAfica.com

The ages of the players are to be grouped into class intervals.

(a) Complete the following table: (2)

Class intervals (ages)	Frequency	Cumulative frequency
$16 \le x < 20$		
$20 \le x < 24$		
$24 \le x < 28$		
$28 \le x < 32$		
$32 \le x < 36$		

(b) On the diagram provided below, draw a cumulative frequency curve for this data. (6)



(c) Use your graph to read off approximate values for the quartiles. (3)
[11]

QUESTION 2

(a) Complete the table and then use the table to calculate the standard deviation. (5)

Class intervals	Frequency (f)	Midpoint (m)	$f \times m$	$m - \bar{x}$	$(m-\bar{x})^2$	$f \times (m-\bar{x})^2$
$20 \leq x < 24$	3	22				
$24 \leq x < 28$	9	26				
$28 \leq x < 32$	8	30				
$32 \leq x < 36$	3	34				
			$\bar{x} =$			

(b) Hence calculate the standard deviation using the table. (2)

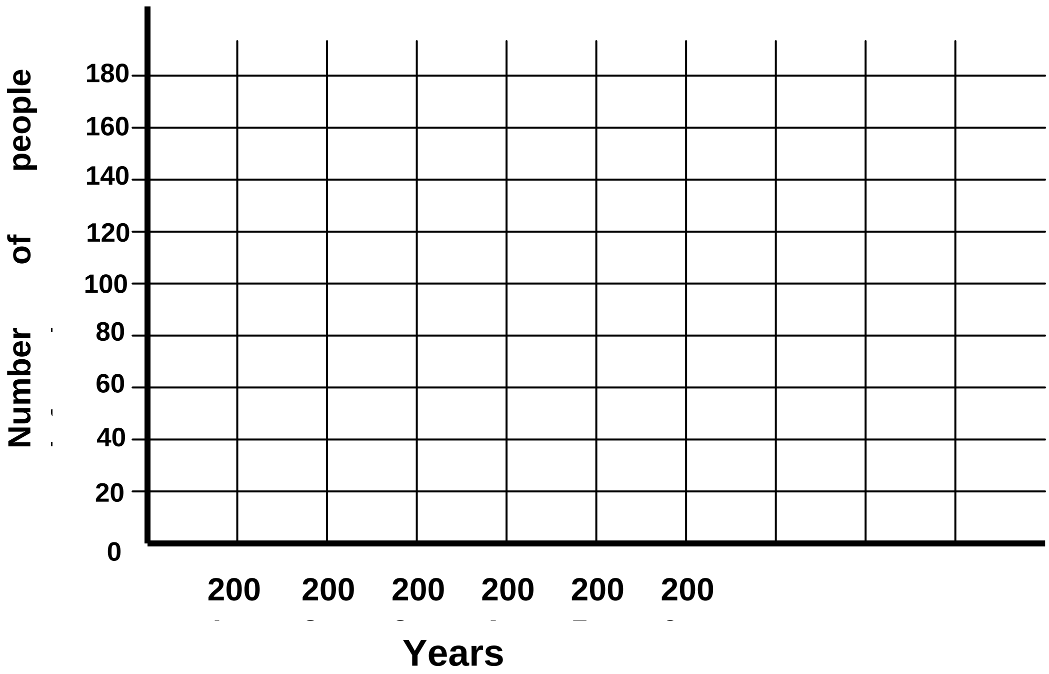
(c) Now use your calculator to verify your answer. (2)
[9]

QUESTION 3

The table below represents the number of people infected with malaria in a certain area from 2001 to 2006:

YEAR	NUMBER OF PEOPLE INFECTED
2001	117
2002	122
2003	130
2004	133
2005	135
2006	137

(a) Draw a scatter plot to represent the above data. Use the diagram provided below. (2)



(b) Explain whether a linear, quadratic or exponential curve would be a line of best fit for the above-mentioned data. (1)

(c) If the same trend continued, estimate, by using your graph, the number of people who will be infected with malaria in 2008. (1)

[4]

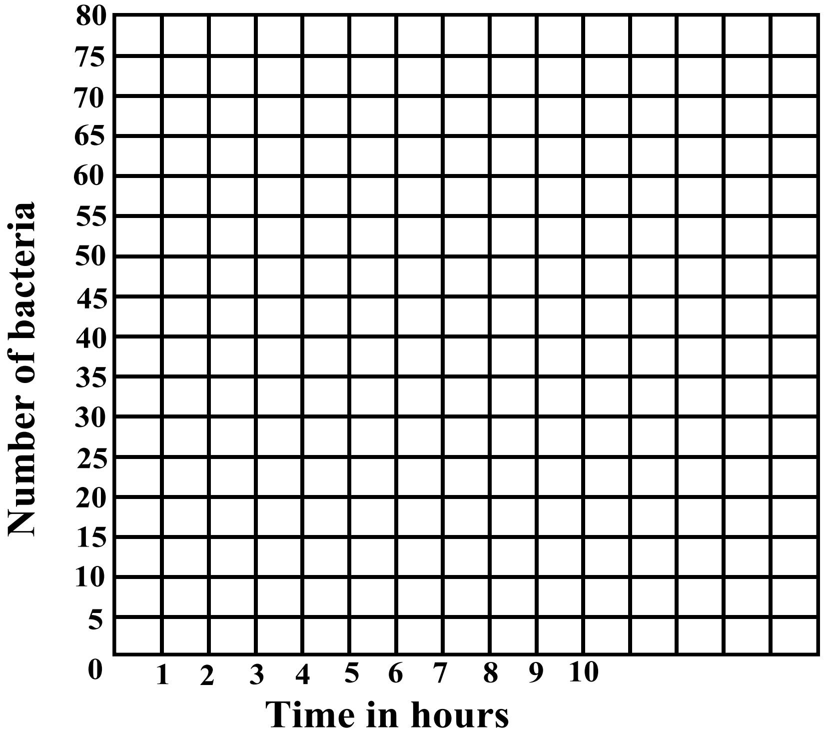
QUESTION 4

A medical researcher recorded the growth in the number of bacteria over a period of 10 hours. The results are recorded in the following table:

Time in hours	0	1	2	3	4	5	6	7	8	9	10
Number of bacteria	5	10	7	13	10	20	30	35	45	65	80

(a) On the diagram provided below, draw a scatter plot to represent this data. (2)

(b) State the type of relationship (linear, quadratic or exponential) that exists between the number of hours and the growth in the number of bacteria. (1)



(3)
[6]

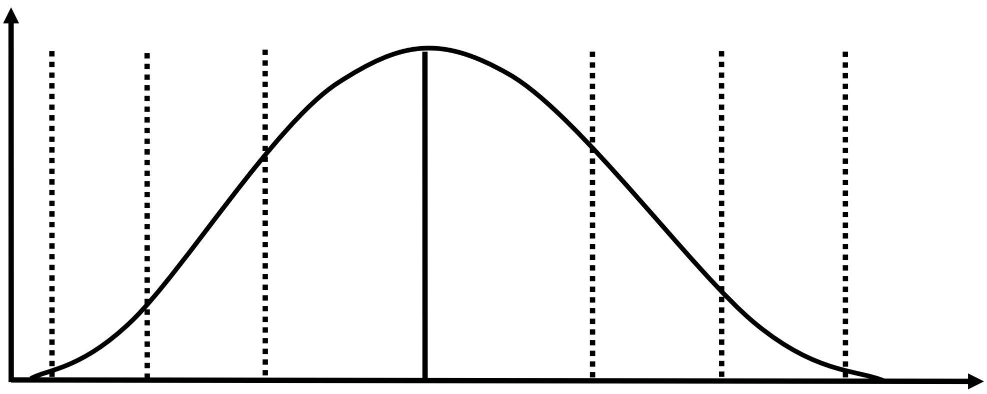
QUESTION 5

The duration of telephone calls made by a receptionist was monitored for a week. The data obtained is represented by the normal distribution curve on the following page. The mean time was 176 seconds with a standard deviation of 30 seconds.

(a) What percentage of calls made was between 146 seconds and 206 seconds in duration? Fill in the necessary information on the graph provided below. (2)

(b) Determine the time interval for the duration of calls for the middle 95% of the data. (2)

(c) What percentage of calls made were in excess of 146 seconds? (2)



[6]

SECTION D: SOLUTIONS AND HINTS TO SECTION A

QUESTION 1

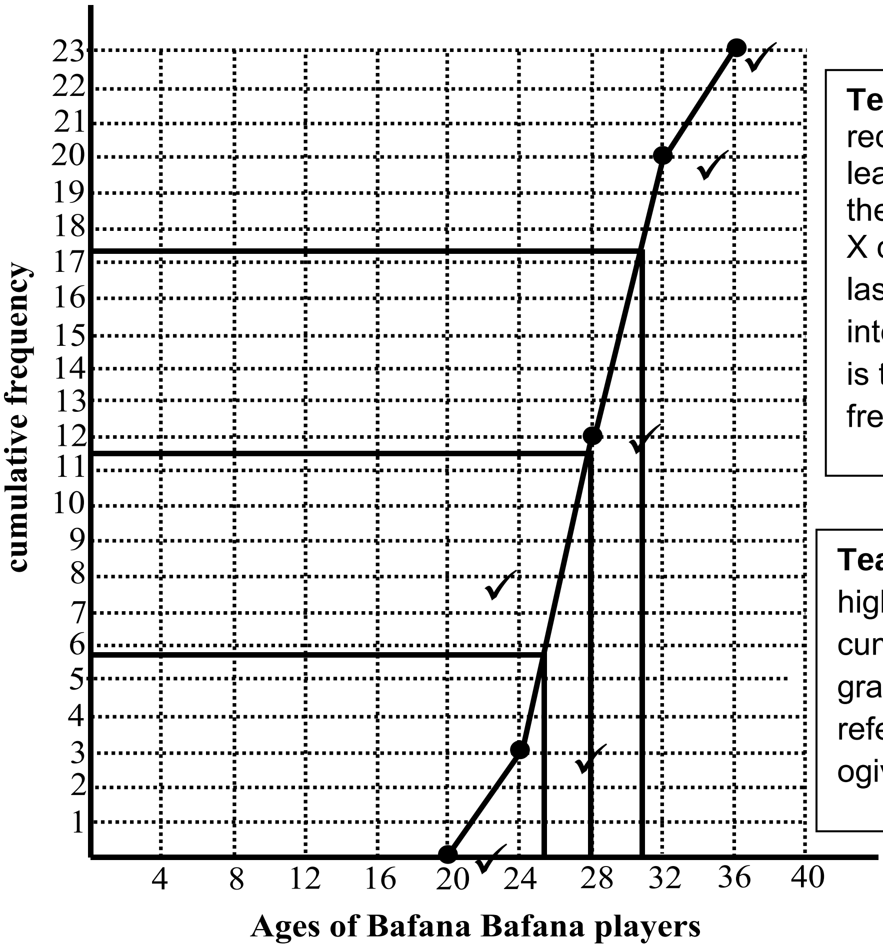
(a)

Class intervals (ages)	Frequency ✓	Cumulative frequency ✓
$16 \le x < 20$	0	0
$20 \le x < 24$	3	3
$24 \le x < 28$	9	12
$28 \le x < 32$	8	20
$32 \le x < 36$	3	23

(2)

(b)

Class intervals (ages)	Frequency	Cumulative frequency	Graph points
$16 \le x < 20$	0	0	$(20 ; 0)$
$20 \le x < 24$	3	3	$(24 ; 3)$
$24 \le x < 28$	9	12	$(28 ; 12)$
$28 \le x < 32$	8	20	$(32 ; 20)$
$32 \le x < 36$	3	23	$(36 ; 23)$



Teacher Note: recommend that learners write down the co-ordinates. X coordinate is the last number in the interval and y value is the cumulative frequency value.

Teacher Note: highlight that a cumulative frequency graph can also be referred to as an ogive.

(6)

(c)

Lower quartile $23 \times \frac{1}{4} = 5.75$ (5,75;25) Therefore Q_1 = 25✓ **Median** $23 \times \frac{1}{-} = 11.5$ (11.5;28) Therefore Median = 28✓ **Upper quartile** $23 \times \frac{3}{4} = 17.25$ (17.25;31) Therefore Q_3 = 31 ✓	**Teacher Note:** remind learners that the lower quartile is the 25th percentile. Therefore, multiply the cumulative frequency by a quarter and read off the graph to determine the y-value. Similarly the median is the 50th percentile. Therefore, multiply the cumulative frequency by a half. The upper quartile is determined by multiplying the cumulative frequency by three quarters (the 75th percentile) (3)

[11]

QUESTION 2

(a)

Class intervals	Frequency (f)	Midpoint (m)	$f \times m$ ✓	$m-\bar{x}$ ✓	$(m-\bar{x})^2$ ✓	$f \times (m-\bar{x})^2$ ✓
$20 \leq x < 24$	3	22	66	–5,9	34,81	104,43
$24 \leq x < 28$	9	26	234	–1,9	3,61	32,49
$28 \leq x < 32$	8	30	240	2,1	4,41	35,28
$32 \leq x < 36$	3	34	102	6,1	37,21	111,63
			$\bar{x} = \frac{642}{23} = 27,9$ ✓			$\sum f \times (m-\bar{x})^2 = 283,83$

(5)

Teacher Note: emphasise that mean can be calculated by multiplying the midpoint of the interval and relative frequency.

(b) $\text{SD} = \sqrt{\frac{\sum f.(x-\bar{x})^2}{23}} = \sqrt{\frac{283,83}{23}} = 3,5$	✓✓ (2)

<table>
<tr>
<td>
CASIO fx-82ES PLUS:
MODE
2 : STAT
1 : 1 – VAR
SHIFT SETUP
3: STAT (you need to scroll down to get this function)
1: ON
Enter the midpoints:
22= 26= 30= 34=
Enter the frequencies:
3= 9= 8= 3=
AC SHIFT 1
4: VAR
3 : $x\sigma n$ =
The answer will read:3,5

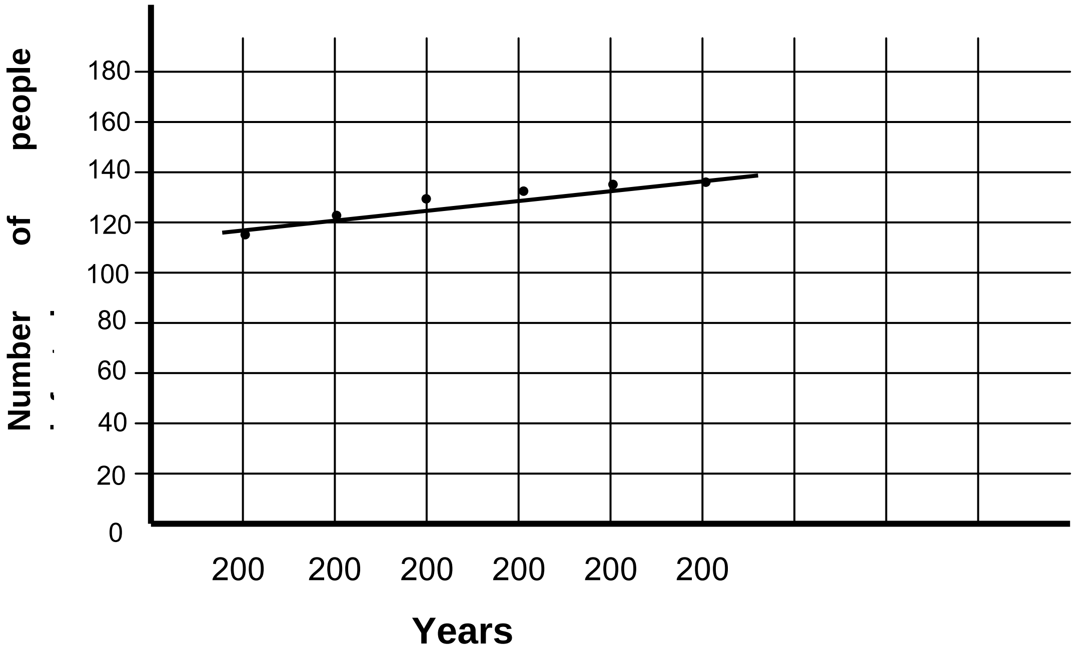
SHARP DAL:

MODE 1=
Enter data:
22 STO 3 M+
26 STO 9 M+
30 STO 8 M+
34 STO 3 M+
RCL 6 to get 3,5
</td>
<td>✓✓ (2)</td>
</tr>
</table>

[9]

QUESTION 3

(a)



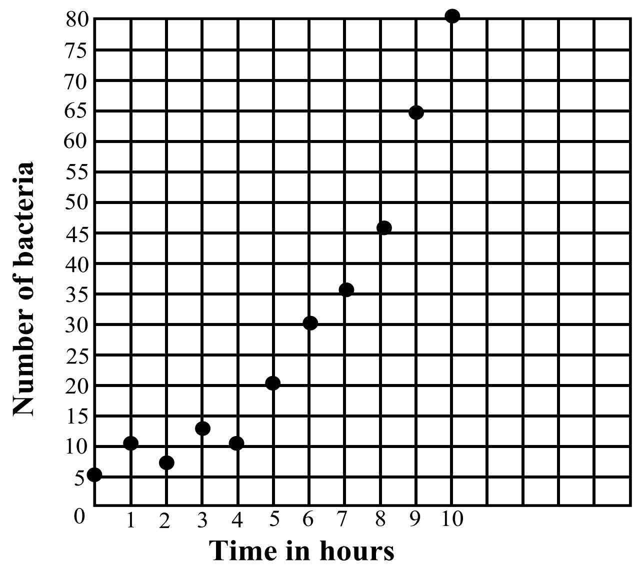
(2)

b. Linear	✓
c. Approx. 145 people	✓

[4]

QUESTION 4

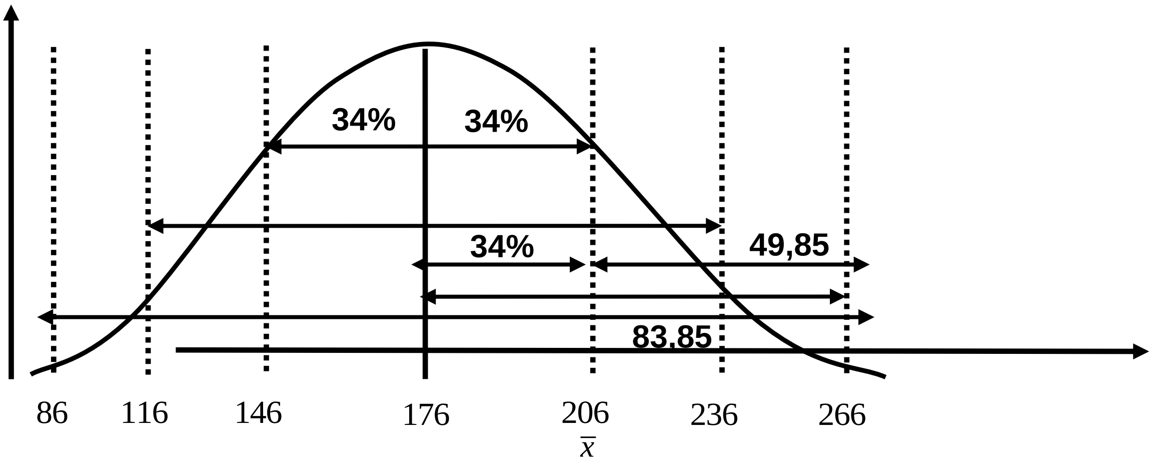
✓✓



b. Quadratic or exponential	✓ **Teacher Note:** remind learners that if ever asked to describe or state the relationship, they should answer by identifying the function, e.g. linear, quardratic, exponential.

[3]

QUESTION 5



One standard deviation interval:

$(\overline{x}-s\,;\,\overline{x}+s)$

$=(176-30;176+30)$

$=(146;206)$

Two standard deviation intervals:

$(\overline{x}-s\,;\,\overline{x}+s)$

$=(176-2(30);176+2(30))$

$=(116;236)$

Three standard deviation intervals:

$(\overline{x}-s\,;\,\overline{x}+s)$

$=(176-3(30);176+3(30))$

$=(86;266)$

a. The interval between 146 seconds and 206 seconds lies between one standard deviation of the mean. For the normal distribution, approximately 68% of the data lies between one standard deviation of the mean.	✓✓
b. The middle 95% of the data for a normal distribution lies between two standard deviations on either side of the mean. The middle 95% of the calls will be between 116 and 236 seconds.	✓✓
c. Approximately 34% of the calls are between 146 and 176 seconds. Another 49, 85% of the calls are in excess of 176 seconds. Therefore, in total, approximately 84% of the calls are in excess of 146 seconds.	✓✓ **[6]**

SECTION C: HOMEWORK

QUESTION 1

The ages of the final 23 players selected by coach Oscar Tabarez to play for Uruguay in the 2010 FIFA World Cup are provided below.

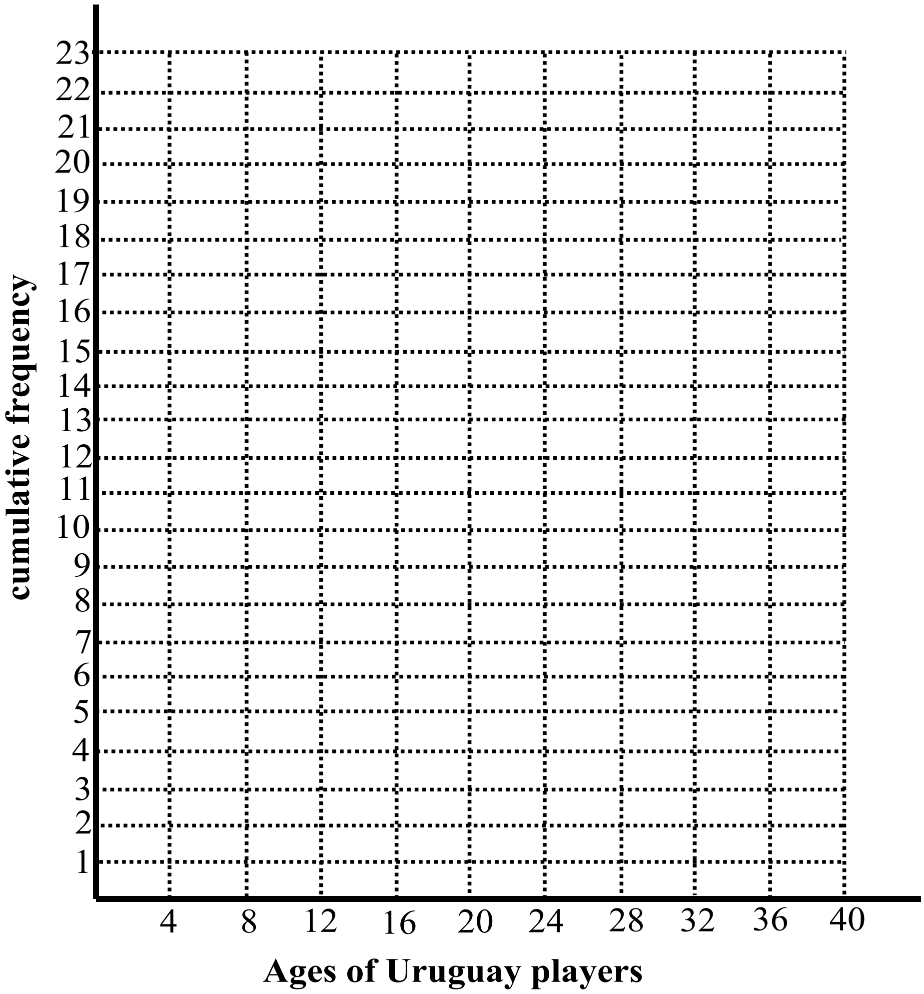
Position	Player	Age
1	Fernando Musiera	23
2	Diego Lugano (captain)	29
3	Diego Godin	24
4	Jorge Fucile	25
5	Walter Gargano	25
6	Andres Scotti	35
7	Edinson Cavani	23
8	Sebastian Eguren	29
9	Luis Suarez	23
10	Diego Forlan	31
11	Alvaro Perreira	25
12	Juan Castillo	32
13	Sebastian Abreu	33
14	Nicolas Lodeira	23
15	Diago Perez	30
16	Maxi Perrreira	26
17	Ignacio Gonzales	28
18	Egidio Arevalo Rios	27
19	Sebastian Fernandes	25
20	Mauricio Victorino	27
21	Alvaro Fernandez	24
22	Martin Caceres	23
23	Martin Silva	27

Source: *www.2010 Fifa World Cup:final squads – MediaClubSouthAfica.com*

(a) Complete the following table: (2)

Class intervals (ages)	Frequency	Cumulative frequency
$16 \le x < 20$		
$20 \le x < 24$		
$24 \le x < 28$		
$28 \le x < 32$		
$32 \le x < 36$		

(b) On the diagram provided below, draw an ogive representing the above data. (6)



(c) Use your graph to read off approximate values for the quartiles. (3)
[11]

QUESTION 2

(a) Complete the table and then use the table to calculate the standard deviation. (5)

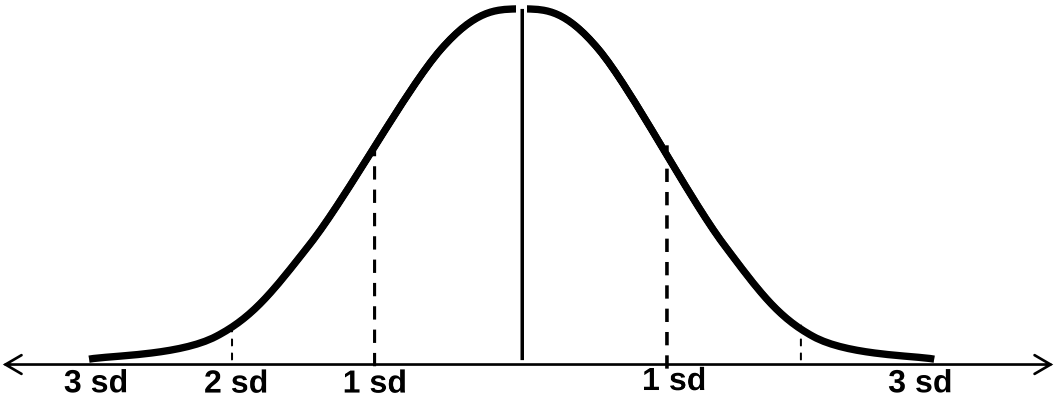
Class intervals	Frequency (f)	Midpoint (m)	$f \times m$	$m-\bar{x}$	$(m-\bar{x})^2$	$f \times (m-\bar{x})^2$
$20 \le x < 24$	3	22				
$24 \le x < 28$	9	26				
$28 \le x < 32$	8	30				
$32 \le x < 36$	3	34				
			$\bar{x} =$			

(b) Hence calculate the standard deviation using the table. (2)

(c) Now use your calculator to verify your answer. (2)
[9]

QUESTION 3

After protracted union protests, a company analysed its salary structure for employees. They found that the salaries were symmetrically distributed with a mean of R8 850 per month and a standard deviation of R2 950 per month. Research indicated that if the monthly salary was below R3 000, the employee would not maintain an acceptable quality of life.



(a) Estimate the percentage of employees who will struggle to maintain an acceptable quality of life. (2)

(b) Estimate the percentage of employees who earn more than R11 800 per month. (1)

(c) Do you think that the company has a fair salary structure? Use the given data to motivate your answer. (1)

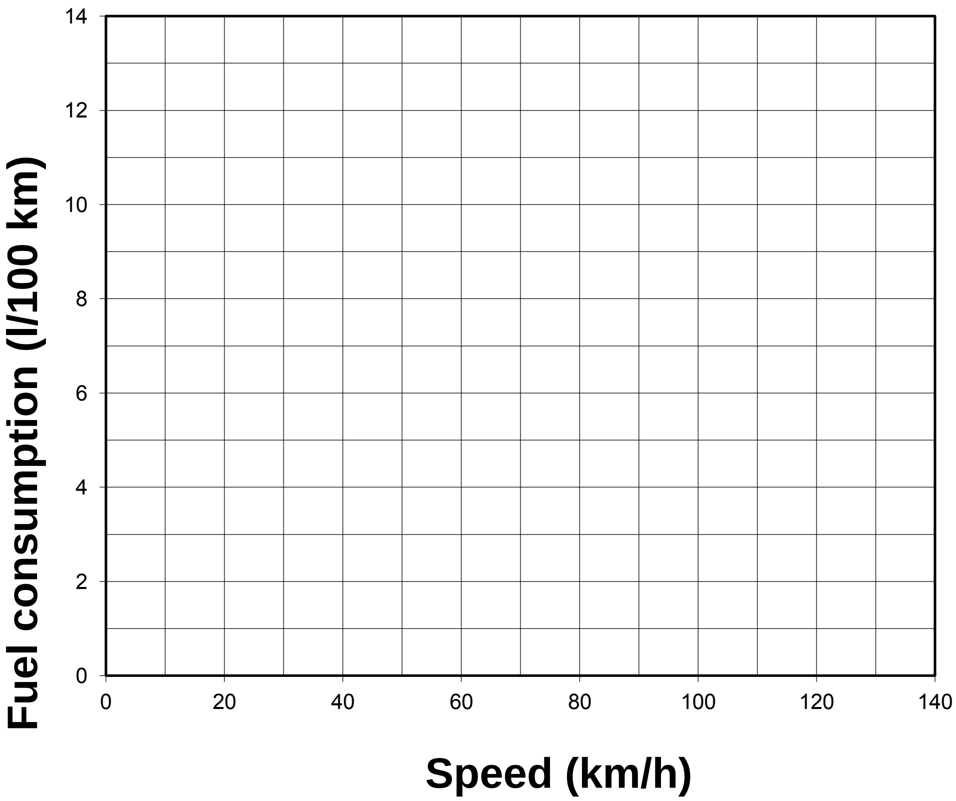
[4]

QUESTION 4

A motor company did research on how the speed of a car affects the fuel consumption of the vehicle. The following data was obtained:

Speed in km/h	60	75	115	85	110	95	120	100	70
Fuel consumption in ℓ/100 km	11,5	10	8,4	9,2	7,8	8,9	8,8	8,6	10,2

(a) Represent the data as a scatter plot on the diagram provided. (3)



(b) Suggest whether a linear, quadratic or exponential function would best fit the data. (1)

(c) What advice can the company give about the driving speed in order to keep the cost of fuel to a minimum? (2)

[6]

SECTION D: HOMEWORK SOLUTIONS

QUESTION 1

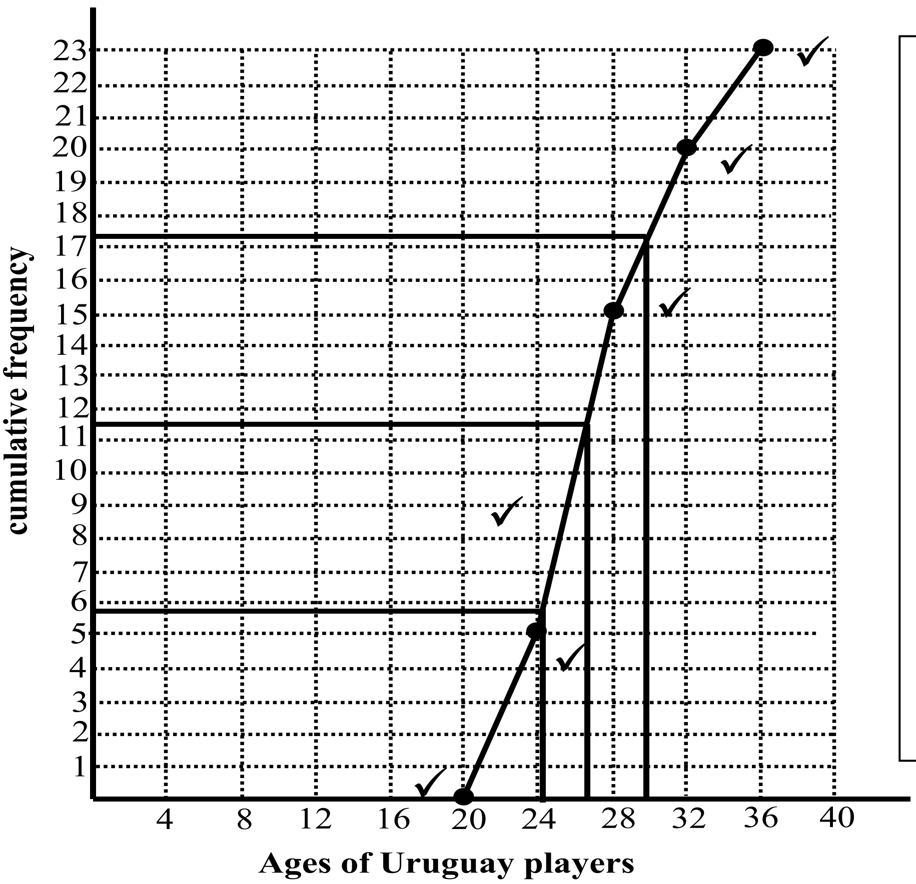
(a)

Class intervals (ages)	Frequency ✓	Cumulative frequency ✓
$16 \le x < 20$	0	0
$20 \le x < 24$	5	5
$24 \le x < 28$	10	15
$28 \le x < 32$	5	20
$32 \le x < 36$	3	23

(2)

(b)

Class intervals (ages)	Frequency	Cumulative frequency	Graph points
$16 \le x < 20$	0	0	$(20;0)$
$20 \le x < 24$	5	5	$(24;5)$
$24 \le x < 28$	10	15	$(28;15)$
$28 \le x < 32$	5	20	$(32;20)$
$32 \le x < 36$	3	23	$(36;23)$



Teacher Note:
Explain to learners that the above table is not required as marks are not given for the column with the coordinates. However, it is a good idea to determine the coordinates carefully before plotting. A mark is given to each point plotted correctly, and one mark given for correct shape of the ogive.

(6)

(c)

Lower quartile $23 \times \frac{1}{4} = 5{,}75$ Therefore $Q_1 = 24$	✓
Median $23 \times \frac{1}{2} = 11.5$ Therefore Median = 26	✓
Upper quartile $23 \times \frac{3}{4} = 17.25$ Therefore $Q_3 = 30$	✓ (3)

[11]

QUESTION 2

(a)

Class intervals	Frequency (f)	Midpoint (m)	$f \times m$ ✓	$m - \bar{x}$ ✓	$(m-\bar{x})^2$ ✓	$f \times (m-\bar{x})^2$ ✓
$20 \le x < 24$	5	22	110	-5	25	125
$24 \le x < 28$	10	26	260	-1	1	10
$28 \le x < 32$	5	30	150	3	9	45
$32 \le x < 36$	3	34	102	7	49	147
			$\bar{x} = \frac{622}{23} = 27$ ✓			$\sum f \times (m-\bar{x})^2 = 327$

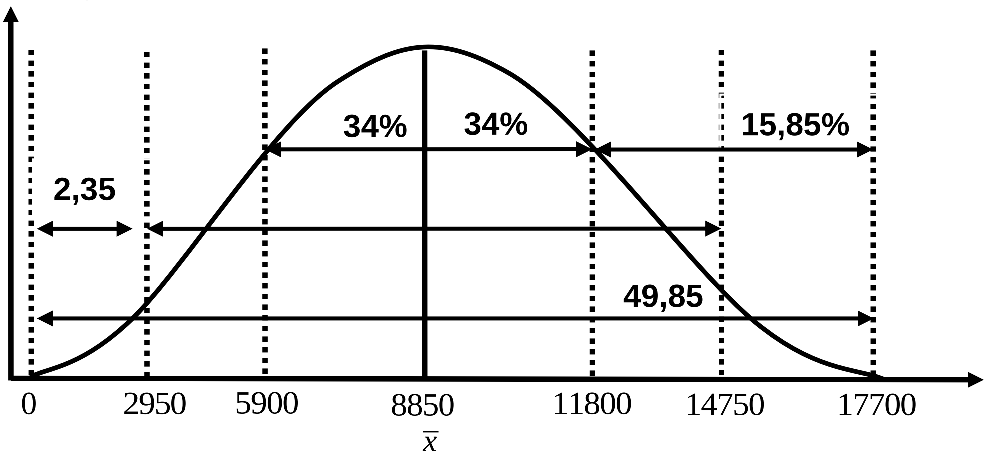
(5)

(b) $\text{SD} = \sqrt{\frac{\sum f.(x-\bar{x})^2}{23}} = \sqrt{\frac{327}{23}} = 3{,}8$	✓✓ (2)

(c)	**CASIO fx-82ES PLUS:** MODE 2 : STAT 1 : 1 – VAR SHIFT SETUP 3: STAT 1: ON Enter the midpoints: 22= 26= 30= 34= Enter the frequencies: 5= 10= 5= 3= AC SHIFT 1 4: VAR 3 : $x\sigma n$ = The answer will read: 3,8 **SHARP DAL:** MODE 1= Enter data: 22 STO 3 M+ 26 STO 9 M+ 30 STO 8 M+ 34 STO 3 M+ RCL 6 to get 3,8	✓✓ (2)

[9]

QUESTION 3



One standard deviation interval:

$(\bar{x}-s\,;\bar{x}+s)$

$=(8850-2950;8850+2950)$

$=(5900;11800)$

Two standard deviation intervals:

$(\bar{x}-2s\,;\bar{x}+2s)$

$=(8850-2\times2950;8850+2\times2950)$

$=(2950;14750)$

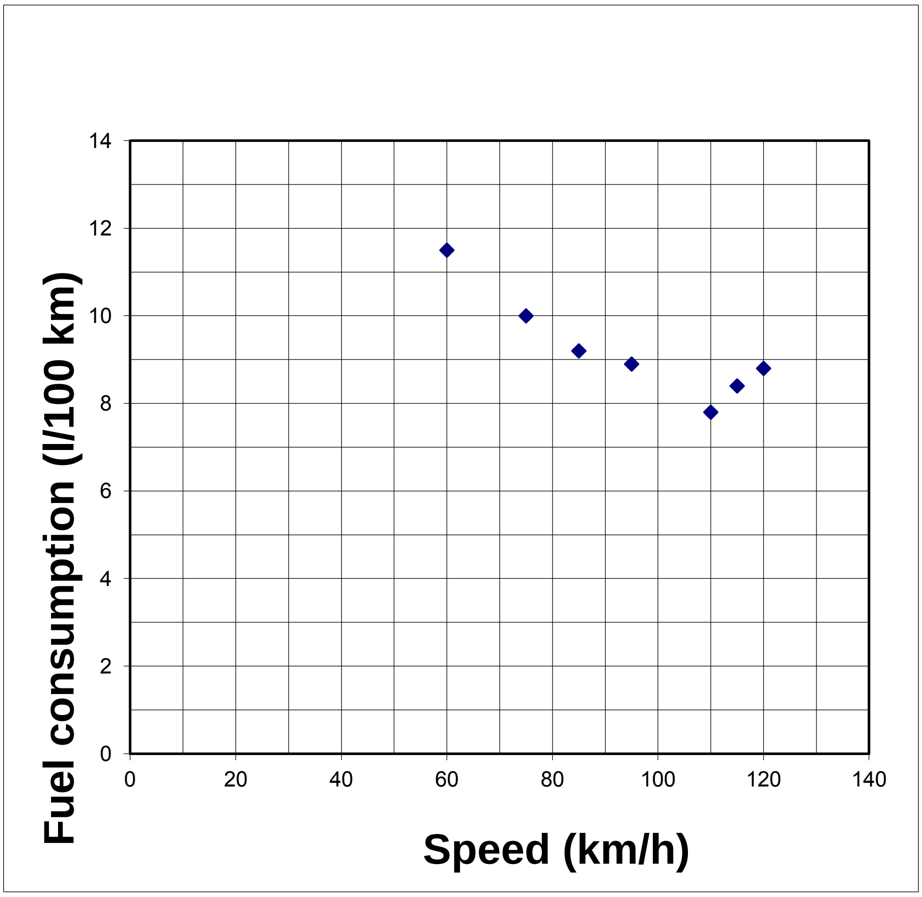
Three standard deviation intervals:

$$(\bar{x} - 3s\,;\, \bar{x} + 3s)$$
$$= (8850 - 3 \times 2950; 8850 + 3 \times 2950)$$
$$= (0; 17700)$$

2%	✓✓	(2)
16%	✓	(1)
No, since there are some employees (less than 2%) earn below R3000,00. These employees will not live an acceptable lifestyle economically. **OR** Yes, there is a fair distribution of salaries since the majority of the employees,i.e. 68% earn a salary between R5 900 and R11 800 per month. Some employees will have more responsibilities or work longer hours and thus must be compensated accordingly. Less than 2% earn below R3000,00.	✓	(1) **[4]**

QUESTION 4

a.



✓
✓
✓

(3)

(b)	Quadratic	✓ (1)
(c)	Based on the quadratic trend the best fuel consumption occurs when the car is driven at 110 km/h. To keep its fuel bill to a minimum, drivers should drive at 110km/h	✓✓ (2)

[6]

SESSION 16.2

TOPIC: TRANSFORMATIONS

Teacher Note: Learners must revise all previous grades work on Transformations. The focus in grade 12 will be on rotations through any angle θ however you may be examined on all the work learned thus far on Transformations.

SECTION A: TYPICAL EXAM QUESTIONS

QUESTION 1

Determine the coordinates of the image of point A(-2; $-\sqrt{3}$) after a clockwise rotation, about the origin through an angle of 210°. [6]

Hint: Be careful with your signs, as this is a clockwise rotation.

QUESTION 2

Consider a square with coordinates P(0; -5), Q(5;0), R(0;5) and S(-5;0) and answer the following questions:

2.1 If P'Q'R'S' is the reflection of PQRS about the line y = -x give the coordinates of P' and R'. (4)

2.2 Calculate, correct to two decimal places, the coordinates of P'' and R'' if P'Q'R'S' is rotated about the origin through an angle of 45° anticlockwise to give P''Q''R''S''. (4)

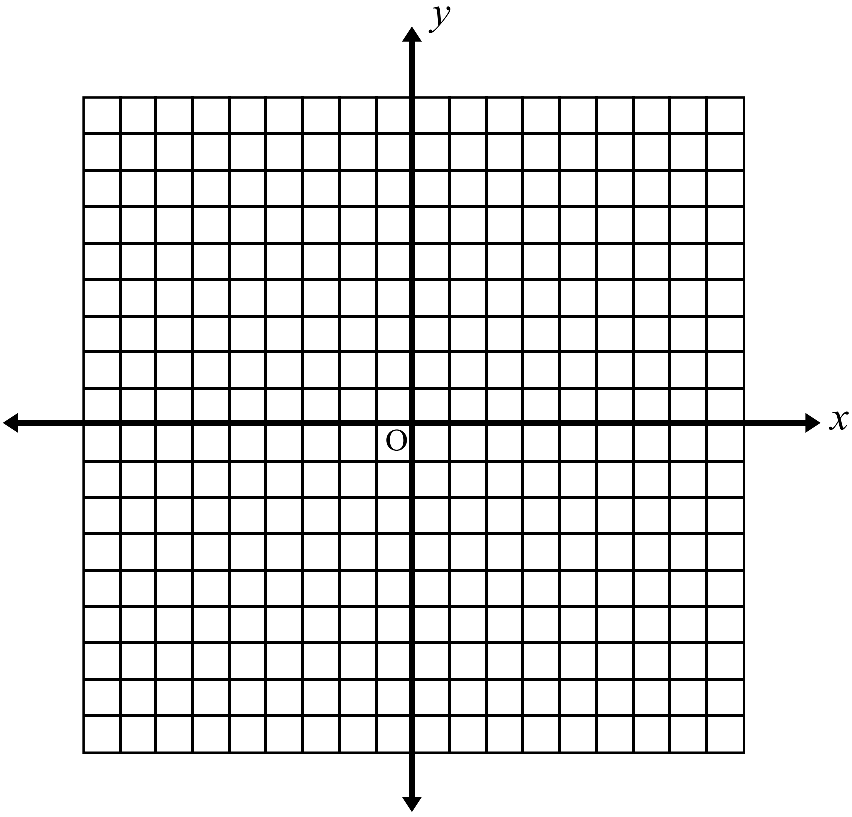
2.3 If S is rotated anticlockwise about the origin through 60° to a new position S''', find the coordinates of S''' without a calculator – leave your answer on surd form. (4)

[24]

QUESTION 3

3.1 On the diagram provided below, draw figure ABCD with the coordinates of the vertices as follows: A (-6;6), B (-4;2), C (-2;6) and D(-4;8). (1)

3.2 On the same diagram, draw the image A'B'C'D' if ABCD is rotated 90°anti-clockwise. Indicate the coordinates of B' and C' (2)



3.3 State the general rule in terms of *x* and *y* of the rotation 90°anti-clockwise, using the notation (x; y) → (1)

3.4 Now draw the image A'B'C'D' if ABCD is transformed under the rule:
(x; y) → ($\frac{1}{2}$x; $\frac{1}{2}$y). (2)

3.5 Write down the value of the following ratio: $\frac{\text{Area ABCD}}{\text{Area A'B'C'D'}}$ (2)

3.6 Draw image A"B"C"D" if A'B'C'D' is rotated 180° clockwise. Indicate the coordinates of A'". (2)

3.7 Draw the image EFGH if A'B'C'D' is transformed under the rule (x, y) → (-x; y). Indicate the coordinates of E. (2)

3.8 Write down the single algebraic rule if ABCD is reflected about the *x*-axis, followed by a translation of 7 units right, followed by an enlargement by a scale factor of 2 units. (4)

[16]

QUESTION 4

A circle with equation $x^2 + y^2 - 2x - 4y = 4$ is rotated 90° anticlockwise about the origin and then enlarged by a scale factor of 2, find the new equation. [6]

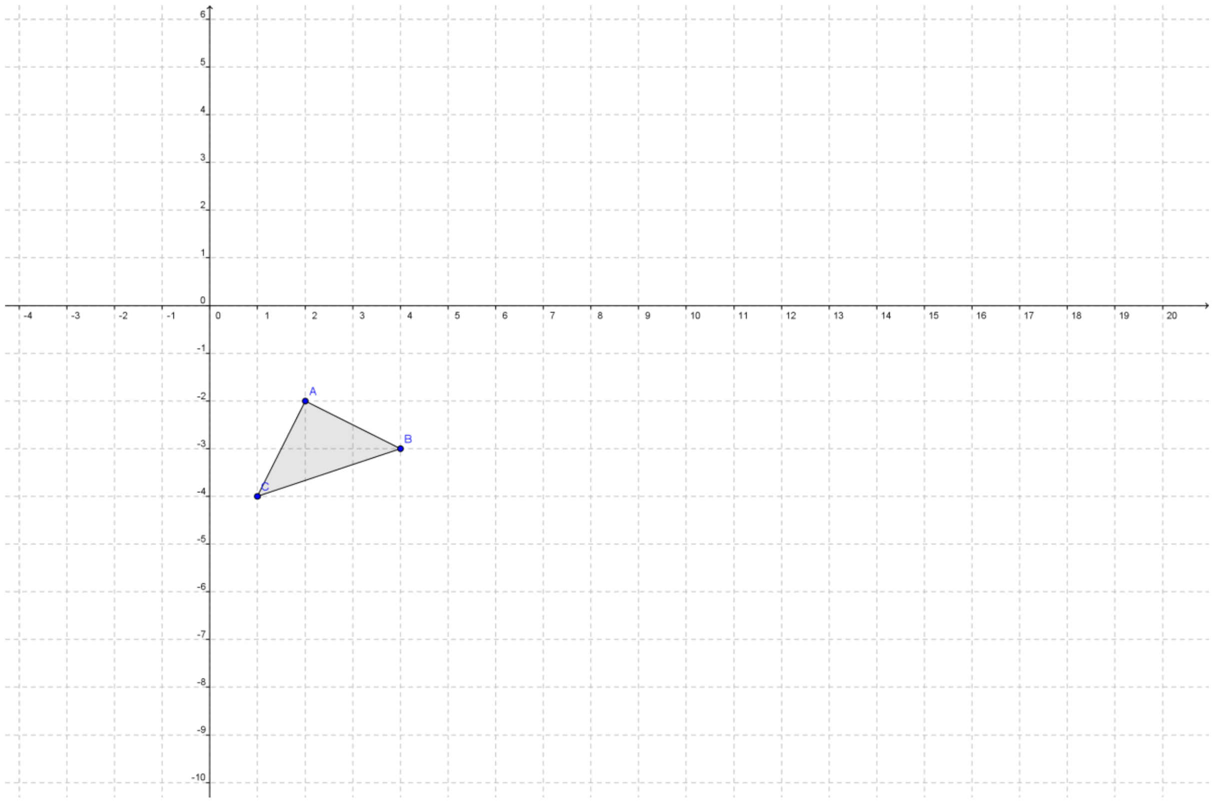
Hint: Rule for rotation through 90° anticlockwise about the origin (x, y) → (-y; x).

QUESTION 5 *(DOE Nov. 2010 P2)*

A transformation T is described as follows:

- A reflection in the *x*-axis, followed by
- A translation of 4 units left and 2 units down, followed by
- An enlargement through the origin by a factor of 2

In the diagram Δ ABC is given with vertices A(2 ; –2), B(4 ; –3) and C(1 ; –4).



5.1 If ΔABC is transformed by T to A'B'C' (in that order), on the same system of axes sketch ΔA'B'C'. Show ALL the steps. (6)

5.2 Write down the general rule for (x; y) under transformation T in the form $(x\ ;\ y) \rightarrow$ (4)

5.3 Calculate the area of ΔA'B'C'. (4)

[14]

QUESTION 6 *(DOE March 2011)*

Consider the point A (– 12 ; 6). The point is reflected about the x-axis to A'

6.1 Write down the coordinates of A'. (1)

6.2 An alternative transformation from A to A' is a rotation about the origin through α°, where α ∈ [0; 90] Calculate α. (6)

[7]

SECTION B: SOLUTIONS TO SECTION A

QUESTION 1

x' = $x_A cos\theta$ - $y_A sin\theta$ = -2cos(-210) – (-$\sqrt{3}$)sin(-210)
= -2cos(-210+360) + 3sin(-210+360) (Add 360 to make angles positive)
= -2cos150 + $\sqrt{3}$sin150
=-2cos(180-30) + 3sin(180-30)
= 2cos30 +$\sqrt{3}$sin30
= 2($\frac{3}{2}$) + 3($\frac{1}{2}$) = $\frac{3\sqrt{3}}{2}$

y' = $y_A cos\theta$ + $x_A sin\theta$ = -$\sqrt{3}$cos(-210) + (-2)sin(-210)
= -$\sqrt{3}$cos150 – 2sin150
= 3cos30 – 2sin30 =$\sqrt{3}$ ($\frac{3}{2}$) -2 ($\frac{1}{2}$) = $\frac{1}{2}$

The image is A'($\frac{3\sqrt{3}}{2}$; $\frac{1}{2}$) **[6]**

QUESTION 2

2.1 Reflection in y=-x: (x; y) → (-y; x)
P'(5; 0) and R'(-5;0) (4)

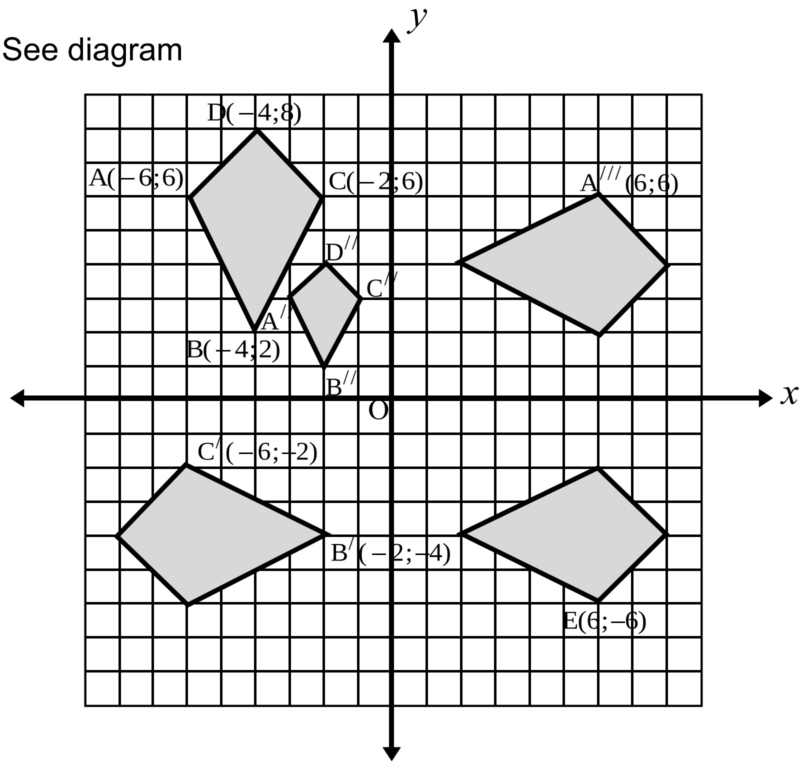
2.2 P''(5cos45;5sin45) ∴ P''(3.54; 3.54) (4)

2.3 S'''(-5cos60; -5sin60)
S'''(-5($\frac{1}{2}$); $-5(\frac{3}{2})$) ∴ S'''($\frac{-5}{2}$; $\frac{-5\sqrt{3}}{2}$) (4)
[12]

QUESTION 3

3.1 &3.2 See diagram (1)

(2)



3.3 $(x\,;y)\rightarrow(-y\,;x)$ (1)

3.4 See diagram (2)

3.5 $\dfrac{\text{Area ABCD}}{\text{Area A}^{//}\text{B}^{//}\text{C}^{//}\text{D}^{//}}=\dfrac{1}{k^2}=\dfrac{1}{(\frac{1}{2})^2}=4$ (2)

3.6 See diagram (2)

3.7 See diagram (2)

3.8

$(x\,;y)\rightarrow(x\,;-y)$

$(x\,;-y)\rightarrow(x+7\,;-y)$

$(x+7\,;-y)\rightarrow(2(x+7)\,;-2y)$

$\therefore(x\,;y)\rightarrow(2x+14\,;-2y)$ (4)

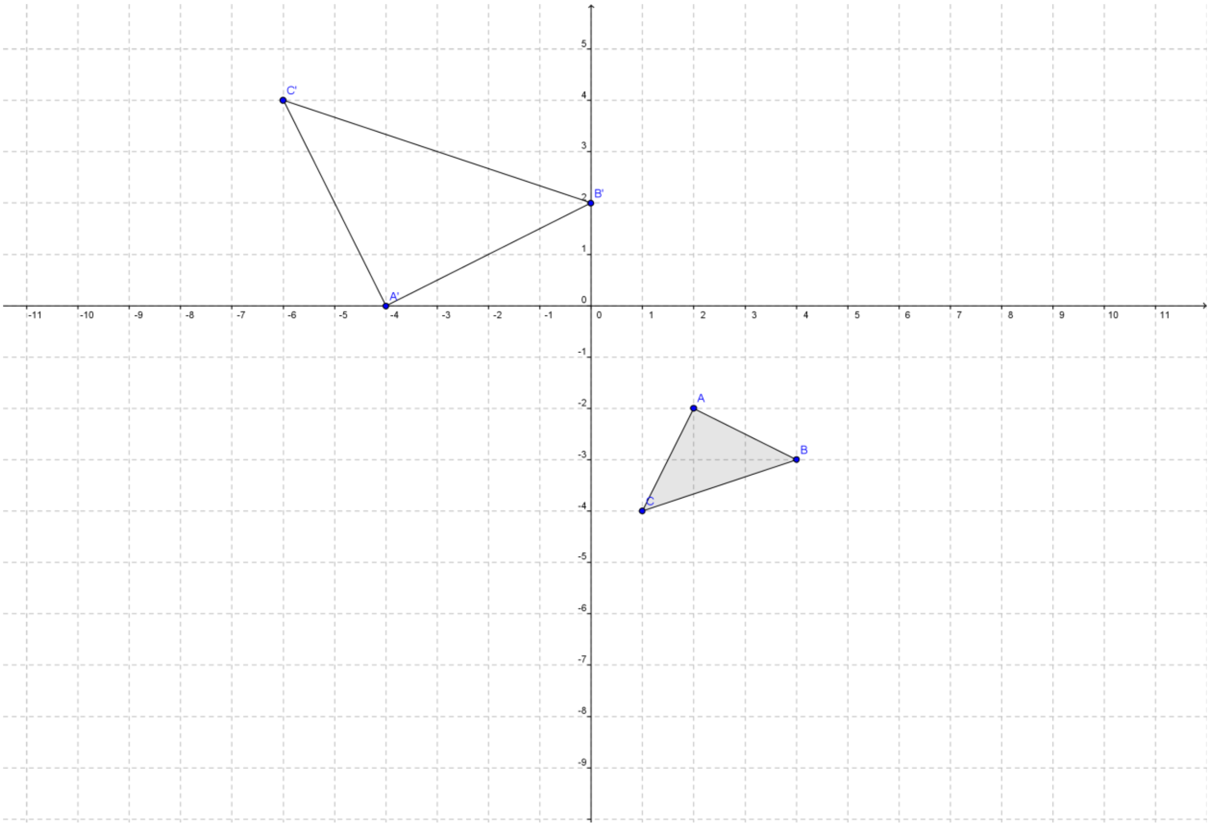
[16]

QUESTION 4

$x^2+y^2-2x-4y=4$
$(x-1)^2+(y-2)^2=9$
∴ the centre is (1;2) and the radius = 3
Rule for rotation through 90°anti-clockwise about the origin; (x; y) → (-y; x)
∴ The centre of the image is (-2;1) The image is then enlarged by a factor of 2
∴ the centre is (-4;2) and the radius = 6.
New Eqn: $(x+4)^2+(y-2)^2=36$ **[6]**

QUESTION 5

5.1



Note:

- If the candidate only draws the correct triangle with labels, **full marks**
- If they plot the points correctly and do not draw the triangle, **max 5 / 6 marks**
- In the 3 sketches, if one vertex of the three is wrong, then 1 / 2 marks for the incorrect sketch, then CA applies.
- If they write down the points and do not plot the points and draw the triangle **max 3 / 6 marks**
- If the vertices are correct but not labelled and the points are joined **max 5 / 6 marks**
- If the vertices are correct, not labelled and not joined **max 4 / 6 marks**
- If a candidate finds a formula first and gets it wrong

Max 1 mark for the formula

Max 2 marks for the calculation of A'B'C' coordinates (CA)

1 mark for plotting 3 vertices

1 mark for completing the triangle and labelling (6)

5.2 $(x; y) \rightarrow (x; -y) \rightarrow (x-4; -y-2) \rightarrow (2x-8; -2y-4)$ (If the candidate gives the answer only-award full marks) (4)

5.3 (Please note: there are several different ways to do this question)

$M_{AC}=2$ and $M_{AB}= -0.5$

Therefore CÂB=90 ($M_{AC} \times M_{AB}=-1$); AB= √5 and AC= √5

Area of ∆ABC = 0.5 (√5)(√5) = 5/2

∴ Area of ∆ A'B'C' = 4 x 5/2 = 10 square units (4)

[14]

QUESTION 6

6.1 A' (-12; -6) (1)

6.2 x' = $x_A cos\alpha$ - $y_A sin\alpha$

-12cosα – 6sinα = -12

-2cosα – sinα =-2 (1)

y' = $y_A cos\alpha$ + $x_A sin\alpha$ = 6cosα – 12 sinα = -6

cosα = 2sinα -1 (2)

substitute (2) into (1)

-2(2sinα -1) – sinα =-2

-4sinα +2 –sinα = -2

-5sinα = -4 sinα =4/5 ……. α =53,13° (6)

[7]

SECTION C: HOMEWORK

QUESTION 1

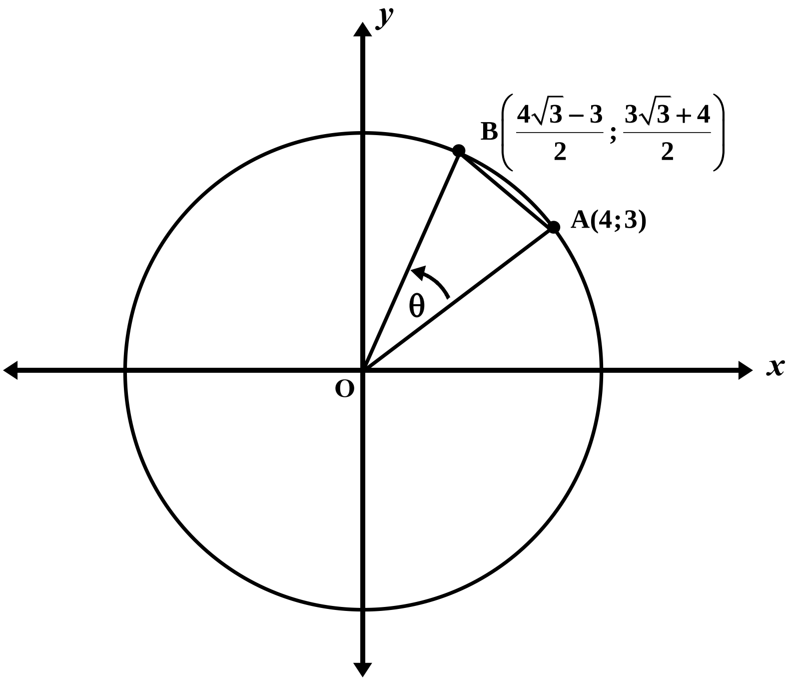
Point P(2; 4) is rotated about the origin through an obtuse angle θ, in an anti-clockwise direction. The image is X(-3 2; y), y<0.

Find:

a) The value of y b) The angle θ

QUESTION 2

In the diagram below, the circle with centre the origin is rotated anti-clockwise about the origin through an angle of θ degrees. Point $A(4\,;3)$ lies on the circle and the image of point A is point B (coordinates are indicated on the diagram).



2.1 Determine the length of the radius of the circle

2.2 Calculate the size of angle θ

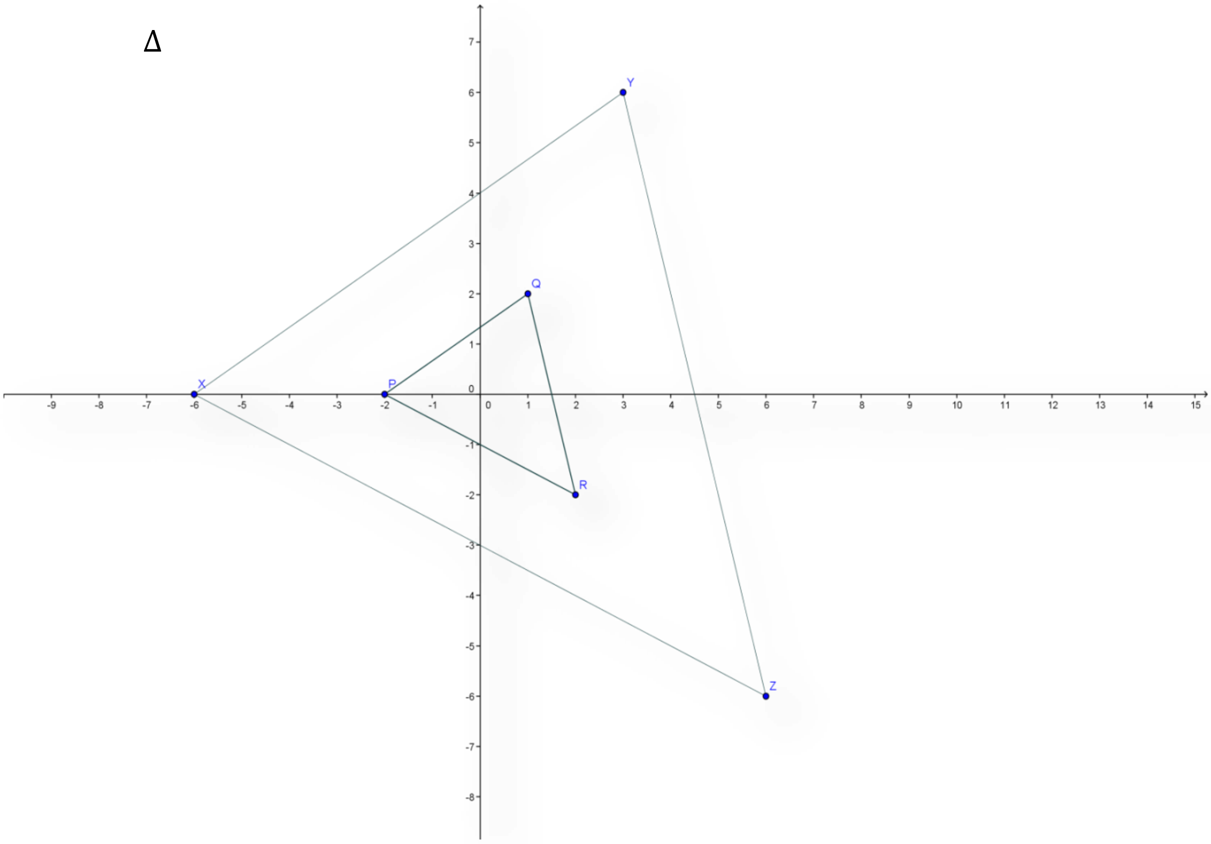
2.3 Hence show that $AB = 5\sqrt{2-\sqrt{3}}$

2.4 Calculate the area of ΔOAB

QUESTION 3

In the diagram below PQR has been enlarged through the origin by a scale factor of 3 to give its image XYZ. Δ

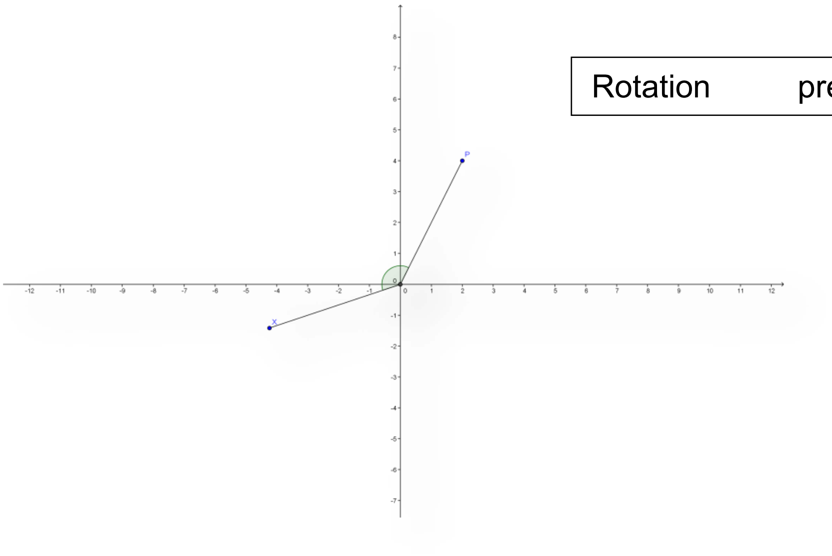
Δ



3.1 Write down the coordinates of XYZ.

3.2 Calculate $\hat{Y}$ (round off your answer to one decimal place)

SECTION D: HOMEWORK SOLUTIONS



Rotation preserved

QUESTION 1

a) OP=OX

$(2-0)^2+(4-0)^2=\left(-3\sqrt{2}-0\right)^2+(y)^2$
20=18 +y^2
∴ y^2 =2
∴ y=$\pm\sqrt{2}$ but y<0 ∴ y= - 2
X($-3\sqrt{2}$; - 2)

b) x' = $x_A cos\theta$ - $y_A sin\theta$ and y' = $y_A cos\theta$ + $x_A sin\theta$
$-3\sqrt{2}$ = $2cos\theta$ - $4sin\theta$ ……(1) - 2 = $4cos\theta$ + $2sin\theta$ …….(2)

Multiply equation (1) by -2 and then add the equations

$6\sqrt{2}$ = $-4cos\theta$ + $8sin\theta$
- 2 = $4cos\theta$ + $2sin\theta$
5$\sqrt{2}$ = $10sin\theta$

$sin\theta$ =$\frac{2}{2}$
∴ θ = 45° but since θ is obtuse θ = 135°

QUESTION 2

2.1
$(4)^2+(3)^2=r^2$
$\therefore r^2=25$
$\therefore r=5$

2.2

$$4\cos\theta - 3\sin\theta = \frac{4\sqrt{3}-3}{2} \text{......A}$$

$$3\cos\theta + 4\sin\theta = \frac{3\sqrt{3}+4}{2} \text{.......B}$$

$$16\cos\theta - 12\sin\theta = 2(4\sqrt{3}-3) \text{......} A\times 4$$

$$9\cos\theta + 12\sin\theta = \frac{3(3\sqrt{3}+4)}{2} \text{.......} B\times 3$$

$$\therefore 25\cos\theta = 2(4\sqrt{3}-3) + \frac{3(3\sqrt{3}+4)}{2}$$

$$\therefore 25\cos\theta = \frac{25\sqrt{3}}{2}$$

$$\therefore \cos\theta = \frac{\sqrt{3}}{2}$$

$$\therefore \theta = 30^\circ$$

2.3

$$AB^2 = (5)^2 + (5)^2 - 2(5)(5)\cos 30^\circ$$

$$\therefore AB^2 = 50 - 50\left(\frac{\sqrt{3}}{2}\right)$$

$$\therefore AB^2 = 50 - 25\sqrt{3}$$

$$\therefore AB^2 = 25(2-\sqrt{3})$$

$$\therefore AB = 5\sqrt{2-\sqrt{3}}$$

2.4

$$\text{Area } \Delta OAB = \frac{1}{2}(5)(5)\sin 30^\circ$$

$$\therefore \text{Area } \Delta OAB = \frac{25}{4} \text{ units}^2$$

QUESTION 3

3.1 X(-6; 0) Y(3, 6) and Z(6; -6)

3.2 Here you will use Analytical geometry to help work out the angles of inclination

Mxy = ⅔ and MYZ = -4

Tanθ = ⅔ tanβ = -4

Θ = 33.69…. β = 104.03….. ∴ α = 75.96…

∴ = 180 – (75.96+33.69) = 70.4°

SESSION 17.1

TOPIC: FUNCTIONS

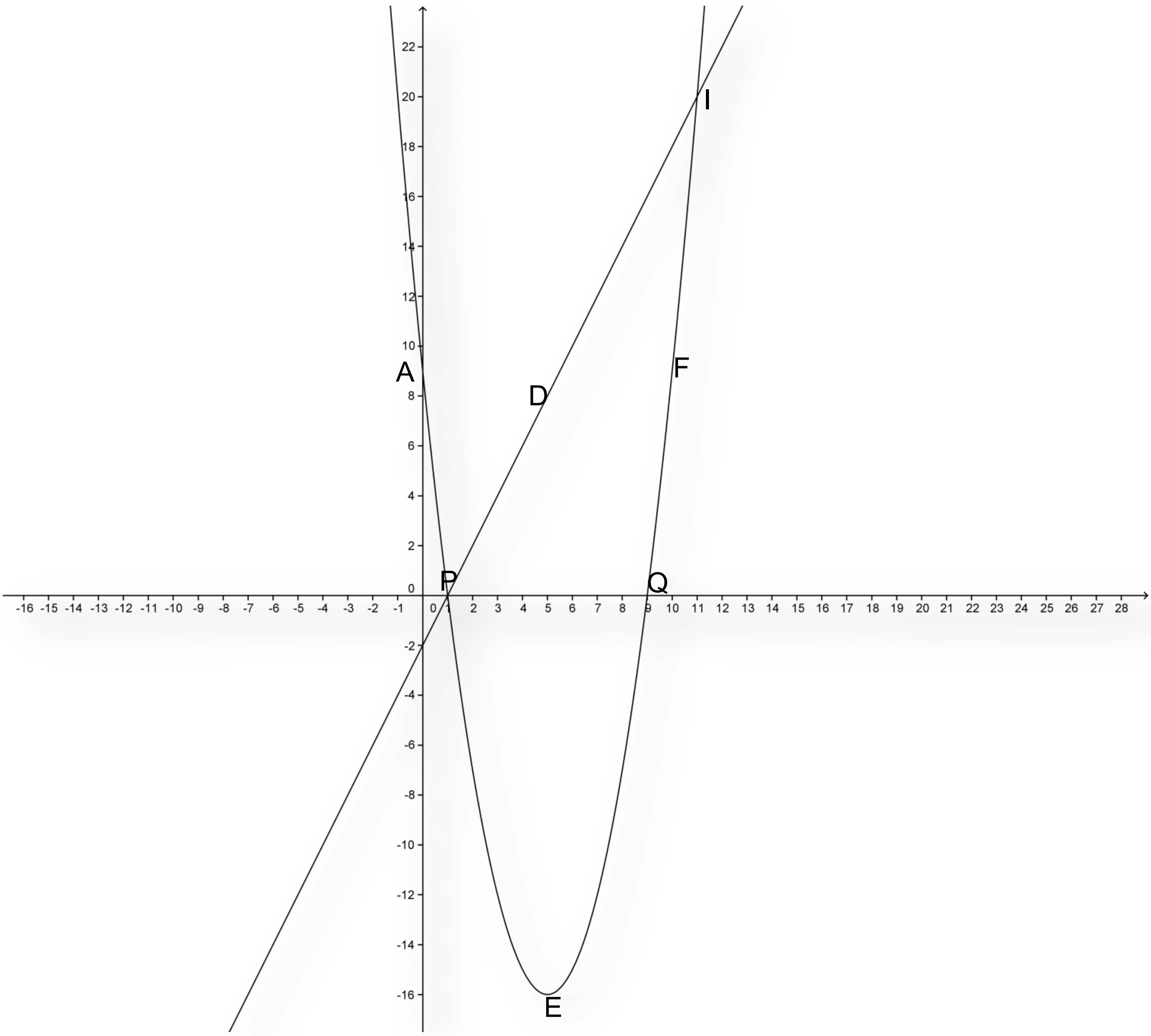
Teacher Note: Learners must be able to sketch all graphs from grades 10 & 11, including their inverses. It is important to understand how to find domain & range, intercepts, turning points, asymptotes and be able to interpret and identify all graphs. Vertical shifts affect the y values and horizontal shifts affect the x values.

SECTION A: TYPICAL EXAM QUESTIONS

QUESTION 1

On the following page, the diagram, which is drawn to scale, shows the graphs of $f(x) = x^2 - 10x + 9$ and $(x) = 2x + c$, where c ∈ ℝ.

The graph of *f* intersects the x-axis at P and Q and the y-axis at A. I and P are points of intersection of *f* and *g.* E is the turning point of *f.* F is a point on *f* such that AF is parallel to the x-axis and D is a point on *g* such that DE is parallel to the y-axis.



1.1 Show that the coordinates of P and Q are (1,0) and (9,0) respectively. (2)

1.2 Show that c=-2 (2)

1.3 Determine the lengths of:

1.3.1 AF (2)

1.3.2 DE (4)

1.4 Determine the equation of p(x) if p is the mirror image of *f* in the line y=0 (1)

1.5 Determine the value of x for which the tangent of *f* will be parallel to *g*. (4)

[15]

QUESTION 2

If $f(x) = \frac{2}{x}$ and $g(x) = x^2 - 2$, determine the value of the following:

2.1 p if $g(-1) = p$ (2)

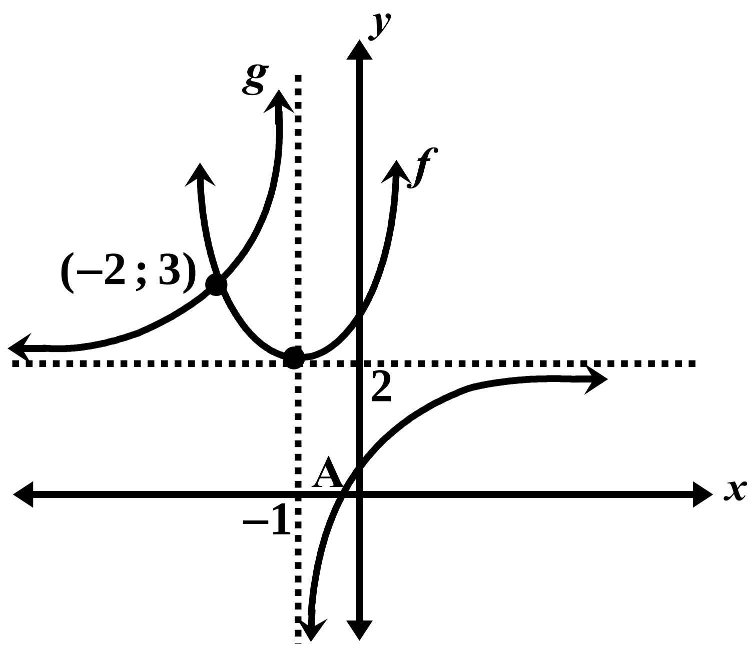
2.2 t if $f(t) = -½$ (2)

2.3 k if $g\left[f\left(\frac{1}{k}\right)\right] = -1$ (5)

[9]

QUESTION 3

In the diagram below, the graphs of f and g are shown. The graphs intersect at (1;3) .
The asymptotes pass through the turning point of the parabola.



3.1 Determine the equation of *f* (3)

3.2 Write down the equations of the asymptotes (2)

3.3 Determine the equation of *g* (3)

3.4 Determine the coordinates of A (3)

3.5 Determine the values of *x* for which $g(x) \leq 0$ (2)

3.6 Determine the equation of the graph formed if *f* is reflected about the *y*-axis (1)

[14]

QUESTION 4

4.1 Sketch the graph of $f(x) = 2^x$ (2)

4.2 Determine the equation of the inverse f^{-1} (2)

4.3 Sketch the graph of the inverse on the same set of axes. (2)

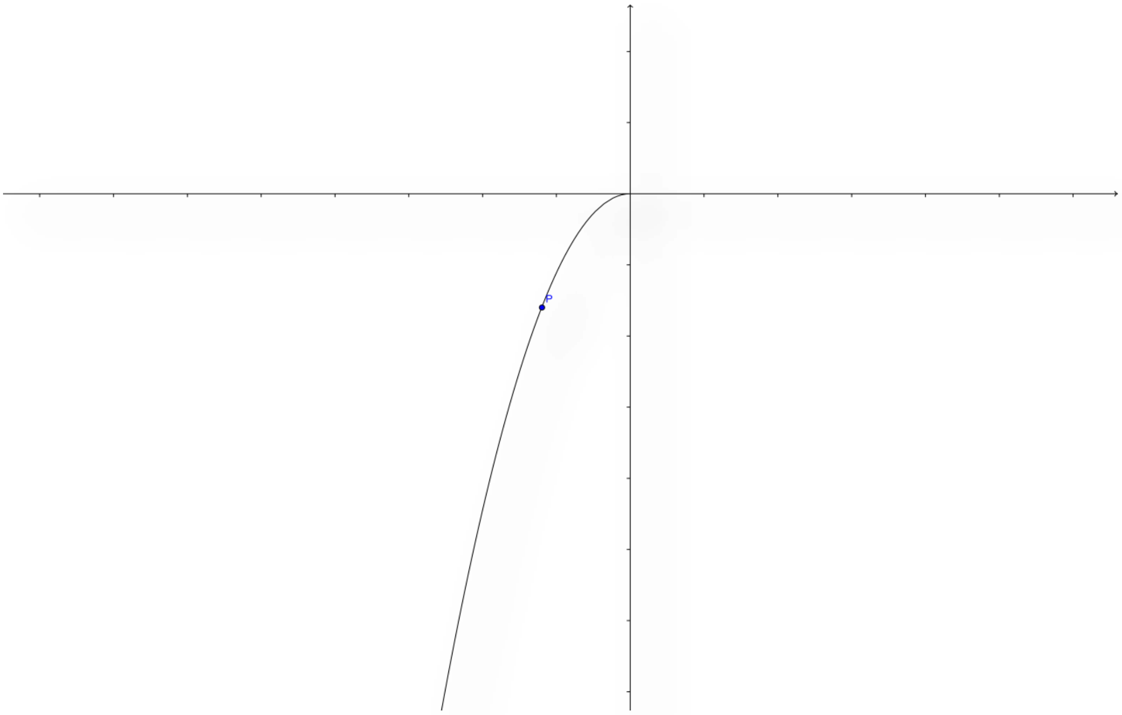
4.4 If the graph of *f* is shifted 1 unit to the right to form the graph *g*, write down the equation of *g*. (1)

4.5 Sketch the graph of *g* on a separate set of axes (2)

[9]

QUESTION 5

The graph of $(x) = ax^2$, $x \leq 0$ is sketched below. The point P(– 6 ; – 8) lies on the graph of *f*.



5.1 Calculate the value of *a* (2)

5.2 Determine the equation of f^{-1} , in the form *y* = … (3)

5.3 Write down the range of f^{-1} (1)

5.4 Draw the graph of f^{-1} on a set of axes. Indicate the coordinates of a point on the graph different from (0 ; 0) (2)

5.5 The graph of *f* is reflected across the line *y* = *x* and thereafter it is reflected across the *x*-axis. Determine the equation of the new function in the form *y* = … (3)

[11]

QUESTION 6

Consider the function $f(x) = \left(\frac{1}{3}\right)^x$

6.1 Is f an increasing or decreasing function? Give a reason for your answer. (2)

6.2 Determine $f^{-1}(x)$ in the form $y = \ldots$ (2)

6.3 Write down the equation of the asymptote of $f(x) - 5$. (1)

6.4 Describe the transformation from f to g if $g(x) = \log_3 x$ (2)
[7]

SECTION B: SOLUTIONS TO SECTION A

QUESTION 1

1.1 x intercepts: P(1; 0) and Q(9; 0) (2)

1.2 graph g intersects f at P ∴ (1; 0) is a point on g
$g(x) = 2x + c$ Substitute point P……. 2(1) +c = 0 ∴ c=-2 (2)

1.3.1 can use symmetry and read off the graph itself OR:
Since AF ∥ x axis point F has co-ordinates (x; 9)
∴ $x^2 - 10x + 9 = 9$
x(x-10) = 0
∴ x=10 or x = 0 ∴ F(10; 9) so AF = 10-0 = 10units (2

1.3.2 Turning point of f: $x = -\frac{b}{2a} = -\frac{(-10)}{2} = 5$
f(5)= 25-50+9 = -16 ∴ E(5;-16)
distance from the x axis to the T.P E is 16 units
D(5;y) lies on the line g(x): g(5) = 2(5) -2 = 8
Distance DE= 8 +16 = 24 (4)

1.4 p(x) = -f(x) = $-x^2 + 10x - 9$ (1)

1.5 Parallel lines have equal gradients m g(x) =2
$f\ (x) = 2x - 10$ =2
2x=12
∴ x =6 (4)
[15]

QUESTION 2

2.1 g(-1) =p
$(-1)^2 - 2 = -1 = p$ (2)

2.2 f(t) = -½
f(t) = $\frac{2}{t} = -\frac{1}{2}$
-t =4 ∴ t= 4 (2)

2.3 $g\left[f\left(\frac{1}{k}\right)\right] = -1$

$\left[f\left(\frac{1}{k}\right)\right] = \frac{2}{\frac{1}{k}} = 2k$

$g(2k) = (2k)^2 - 2 = 4k^2 - 2$

$4k^2 - 2 = -1$

$4k^2 = 1$

$k^2 = \frac{1}{4}$

$\therefore k = \pm\frac{1}{2}$ (5)

[9]

QUESTION 3

3.1

$y = a(x+1)^2 + 2$

$3 = a(-2+1)^2 + 2$

$\therefore 1 = a$

$\therefore f(x) = (x+1)^2 + 2$ (3)

3.2

$x = -1$

$y = 2$ (2)

3.3

$y = \dfrac{a}{x+1} + 2$

$\therefore 3 = \dfrac{a}{-2+1} + 2$

$\therefore 3 = -a + 2$ (3)

$\therefore a = -1$

$\therefore g(x) = \dfrac{-1}{x+1} + 2$

3.4

$0 = \dfrac{-1}{x+1} + 2$

$\therefore 0 = -1 + 2x + 2$

$\therefore -2x = 1$

$\therefore x = -\dfrac{1}{2}$

$\therefore \mathrm{A}\left(-\dfrac{1}{2};0\right)$ (3)

3.5

$g(x) \le 0$

$\therefore -1 < x \le -\dfrac{1}{2}$ (2)

3.6

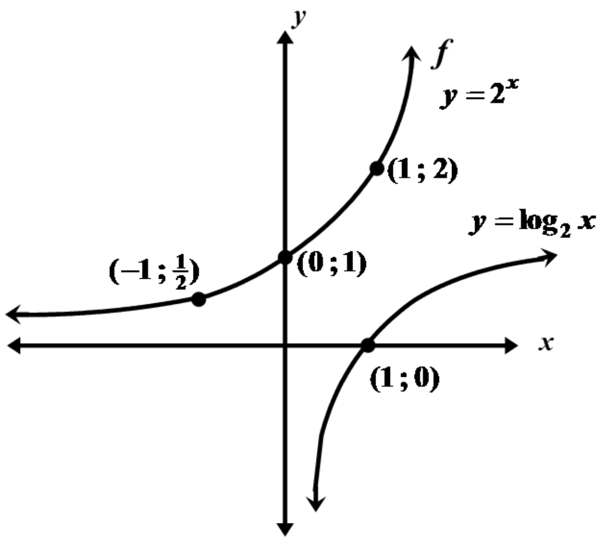
$$y = (x-1)^2 + 2$$

(1)

[14]

QUESTION 4

4.1



(2)

4.2

$y = 2^x$

$\therefore x = 2^y$

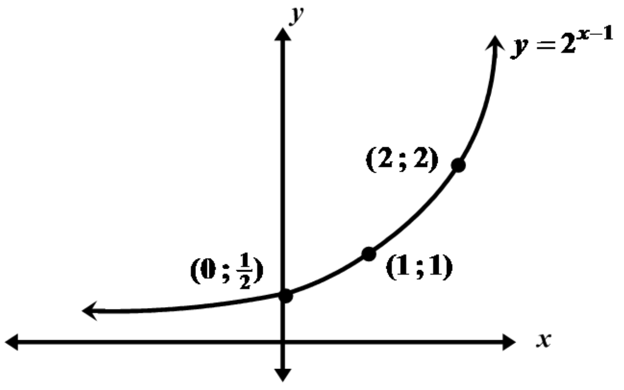
$\therefore y = \log_x 2$

$f^{-1}(x) = \log_x 2$ (2)

4.3 See diagram (2)

4.4 $g(x) = 2^{x-1}$ (1)

4.5



(2)

[9]

QUESTION 5

5.1 $f(x) = ax^2$

$a(-6)^2 = -8$

36a = -8

$\therefore$ a= $-\frac{2}{9}$ (2)

5.2 $y = -\frac{2}{9}x^2$

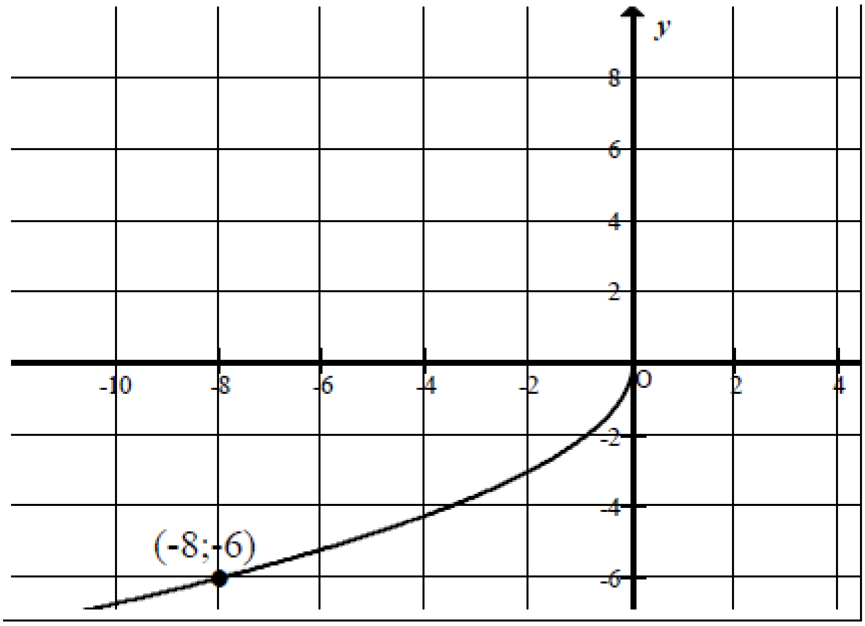
$x = -\frac{2}{9}y^2$

$y^2 = -\frac{9}{2}x$

$y = \pm\sqrt{-\frac{9x}{2}}$, since y≤0 $y = -\sqrt{-\frac{9x}{2}}$ or y = $-3\sqrt{-\frac{x}{2}}$ (3)

5.3 y≤0 (1)

5.4



(2)

5.5 $y = -f^{-1}(x) = -\left(-\sqrt{-\frac{9x}{2}}\right) = \sqrt{-\frac{9x}{2}}$ or $3\sqrt{-\frac{x}{2}}$

(3)

[11]

QUESTION 6

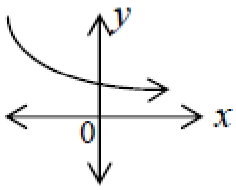
6.1 Decreasing function

Since *0*< *a* < 1 OR As *x* increases, *f(x)* decreases (2)

6.2 $f^{-1}: \quad x = \left(\frac{1}{3}\right)^y$

$y = \log_{\frac{1}{3}} x$

OR



OR

$f^{-1}: \quad x = \left(\frac{1}{3}\right)^y$

$y = -\log_3 x$ (2)

6.3 y=-5 (1)

6.4 Reflection about *y* = *x*.
Reflection about the *x*-axis.
OR
Reflection about the *y*-axis.
Then reflection about the line *y* = *x*.
OR
Reflection about the line *y* = -*x* followed by reflection about the *y*-axis.
OR
Rotation through 90° in a clockwise direction.
OR
Rotation through 90° in an anti-clockwise direction.
Reflection through the origin. (2)
[7]

SECTION C: HOMEWORK

QUESTION 1

Given f(x) = 2x find a simplified expression for: $f(x) + f\left(\frac{1}{x}\right) + \frac{1}{f(x)} + f^{-1}(x)$ [6]

QUESTION 2

Given that a function f satisfies the following conditions :
f(0) =2 , f(-2) = 0 , f'(-1) = 0 and f'(x) > 0 for x ≠ 0
Draw a rough sketch of the graph f. [5]

QUESTION 3

Consider the function $f(x) = (x+1)^2 - 4$

3.1 Draw a neat sketch graph indicating the coordinates of the intercepts with the axes, the coordinates of the turning point and the equation of the axis of symmetry. (6)

3.1.1 Give the range. (2)
[8]

QUESTION 4

Given: $f(x) = a^x$ passing through the point (2; ¼) ; $g(x) = 4x^2$

4.1 Prove that a = ½ (2)

4.2 Determine the equation of $y = f^{-1}(x)$ in the form y = …. (2)

4.3 Determine the equation of y = h(x) where h(x) is the reflection of f(x) about the x-axis (1)

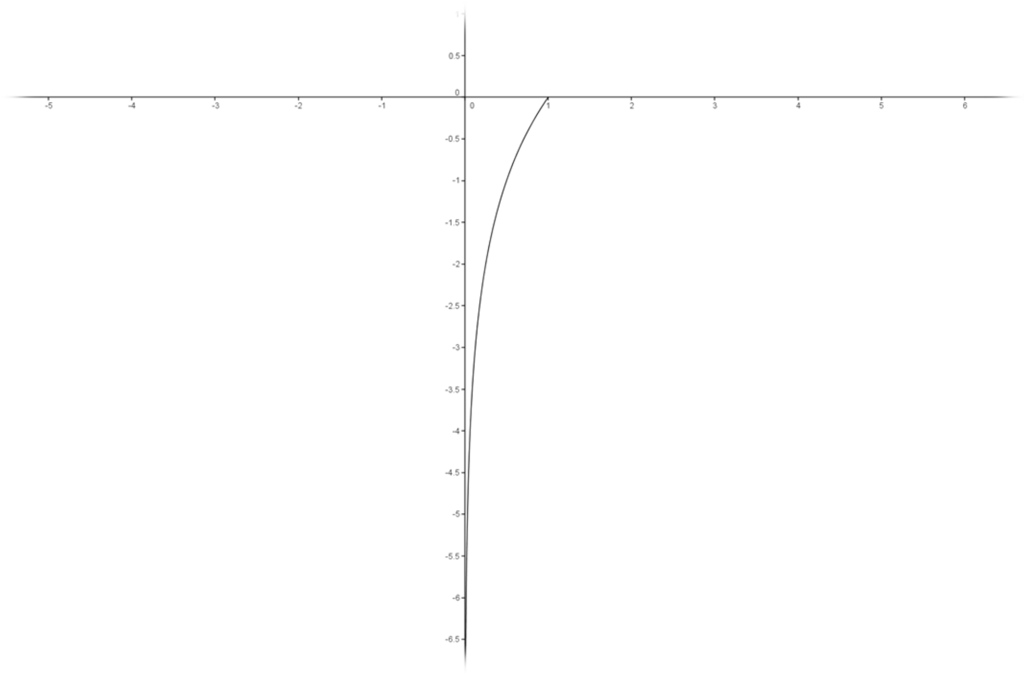
4.4 Determine the equation of the inverse of g in the form y = (2)

4.5 How must the domain of g(x) be restricted so that $g^{-1}(x)$ will be a function? (2)

[9]

QUESTION 5

The graph of g is shown where $g(x) = \log_a x$ where 0<x<1
(½; -1) is a point on g



5.1 Determine the value of a (2)

5.2 Write down the domain of g(x) (2)

5.3 Write the equation of g^{-1} in the form $g^{-1}(x) = ….$ and state the domain (2)

[6]

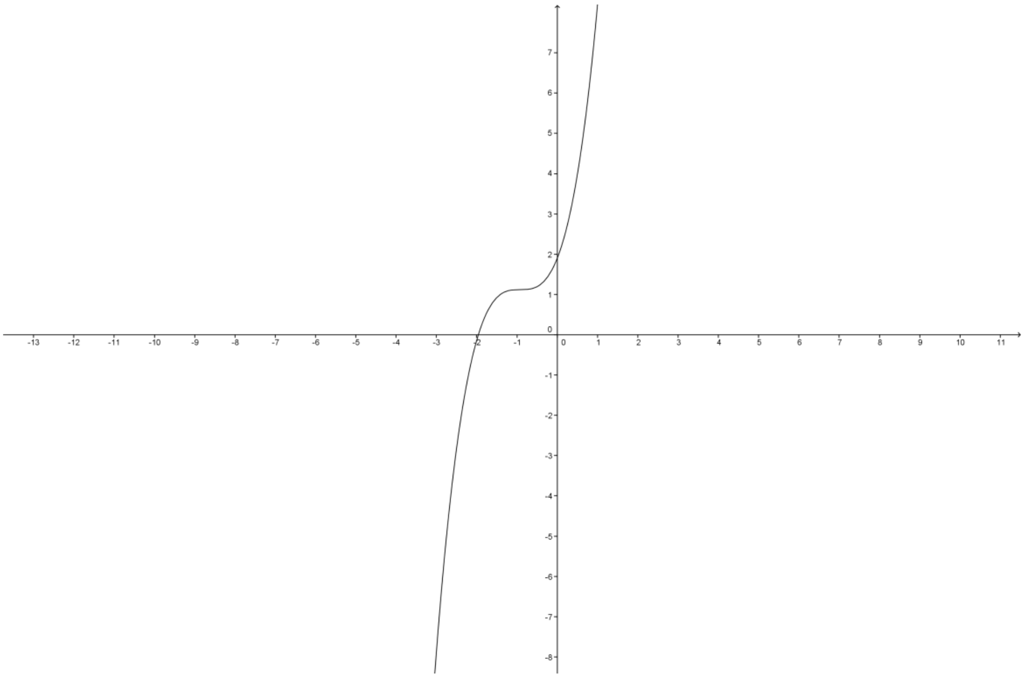
SECTION D: HOMEWORK SOLUTIONS

QUESTION 1

$$f(x) = 2x$$
$$f\left(\frac{1}{x}\right) = 2\left(\frac{1}{x}\right)$$
$$\frac{1}{f(x)} = \frac{1}{2x}$$
$f^{-1}(x) = \frac{1}{2}x$ y=2x swop x and y to find inverse: x = 2y so y =½x
$$f(x) + f\left(\frac{1}{x}\right) + \frac{1}{f(x)} + f^{-1}(x) = 2x + \frac{2}{x} + \frac{1}{2x} + \frac{1}{2}x$$
$$= \frac{5x^2+5}{2x}$$

[6]

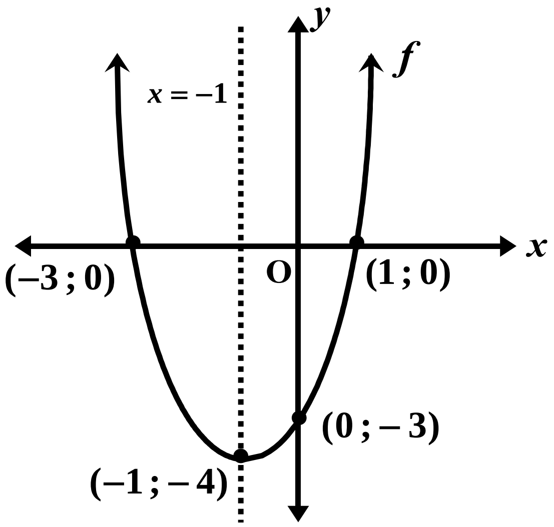
QUESTION 2



[5]

QUESTION 3

3.1



(6)

3.1.1 Range: $y \in [-4\,;\infty)$ (2)
[8]

QUESTION 4

4.1

$$y = a^x$$
$$\therefore \frac{1}{4} = a^2$$
$$\therefore a = \frac{1}{2}$$
(2)

4.2

$$y = \left(\frac{1}{2}\right)^x$$
$$\therefore x = \left(\frac{1}{2}\right)^y$$
$$\therefore y = \log_{\frac{1}{2}} x$$
(2)

4.3

$$y = \left(\frac{1}{2}\right)^x$$
(1)

4.4

$$y = 4x^2$$
$$\therefore x = 4y^2$$
$$\therefore \frac{x}{4} = y^2$$
$$\therefore y = \pm\sqrt{\frac{x}{4}}$$
(2)

4.5 $x > 0$ or $x < 0$ (2)
[9]

QUESTION 5

5.1 g(-½) = -1
$$\log_a \frac{1}{2} = -1$$
$$\therefore a^{-1} = \frac{1}{2}$$
$$\therefore a = 2$$
(2)

5.2 x> 0 and x≠ 1 (NB: The graph of g is only drawn for 0<x<1 but this is not the domain) (2)

5.3 $g^{-1}(x) = 2^x$
$x \in \mathbb{R}, x \neq 0$ (NB: From the log graph x≠ 1 so its' inverse will have y ≠ 1 the value that will make y=1 in $g^{-1}(x)$ is x=0 so it must be excluded from the domain.) (2)
[6]

SESSION 17.2

TOPIC: CALCULUS

LESSON OVERVIEW

1. Introduction session: 10 minutes
2. Typical exam questions:
   Question 1: 10 minutes
   Question 2: 15 minutes
   Question 3: 10 minutes
   Question 4: 15 minutes
   Question 5: 15 minutes
3. Discussion of solutions: 15 minutes

SECTION A: TYPICAL EXAM QUESTIONS

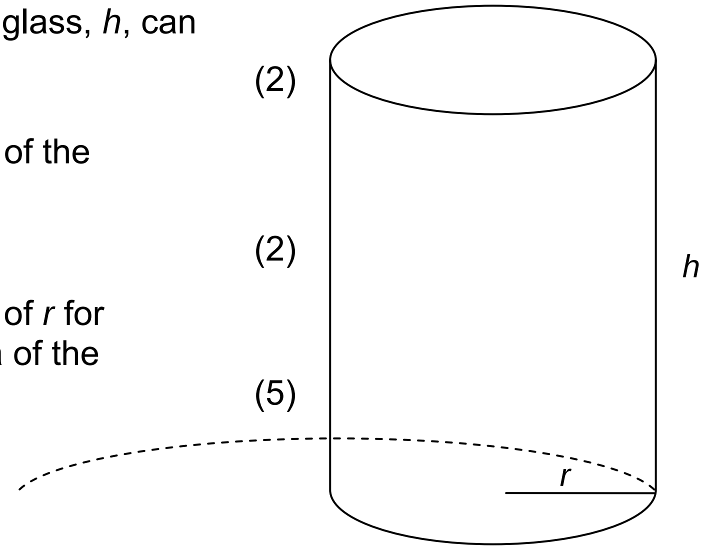
QUESTION 1: 10 minutes

A drinking glass, in the shape of a cylinder, must hold 200 mℓ of liquid when full.

1.1 Show that the height of the glass, *h*, can be expressed as $h = \dfrac{200}{\pi r^2}$. (2)

1.2 Show that the surface area of the glass can be expressed as $S(r) = \pi r^2 + \dfrac{400}{r}$. (2)

1.3 Hence determine the value of *r* for which the total surface area of the glass isa minimum. (5)



[9]

QUESTION 2: 15 minutes

2.1.Differentiate $f(x) = -3x^2$ by first principles. (5)

2.2.If $f(x) = (3x^2 - x)^2$, find $f'(x)$ (4)

2.3.If $y = \frac{2x^2+3x-2}{x+2}$, find $\frac{dy}{dx}$ (3)

2.4.Find $\frac{d}{dx}\left(\frac{4x^3-3x^2}{2\sqrt{x}}\right)$ (4)

[16]

QUESTION 3: 10 minutes

3.1.Evaluate: (3)

$$\lim_{p\to 4}\frac{p^2-5p+4}{4-p}$$

3.2. Leaving your answer with positive exponents, find $f'(x)$ if: (3)

$$f(x)=4x^2-4x+\frac{1}{x^2}-\frac{1}{2}$$

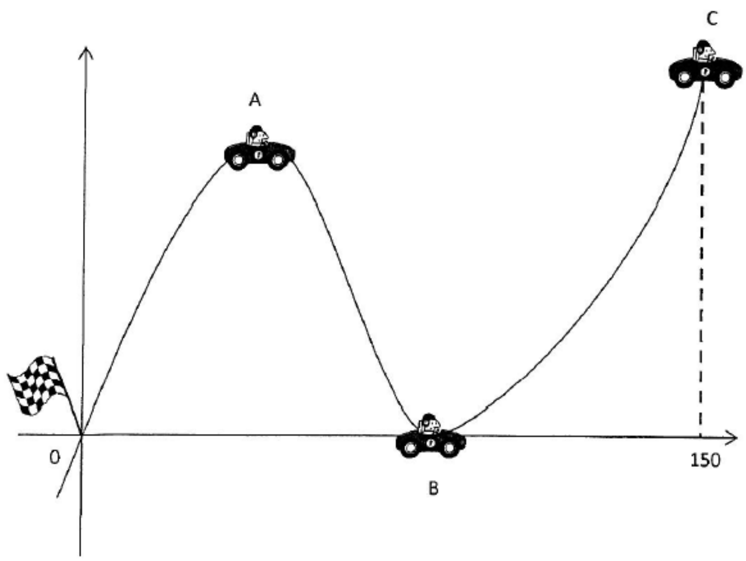
3.3.Evaluate, leaving your answer in surd form, with positive exponents: (4)

$$D_x\left[\frac{5x^2-3x}{\bar{x}}\right]$$

[10]

QUESTION 4: 15 minutes

Part of a rally track follows the path of a cubic curve. A plain view of this section of the track is shown in the diagram below. At a certain instant, cars A and B are at the turning points of the curve. Car C is at the point where x=150. The race starts at the origin, 0.



The function which describes this part of the track is:

$$f(x)=\frac{1}{900}x^3-\frac{1}{5}x^2+9x$$

4.1. Determine the coordinates of cars A and B (6)

4.2. Find the average gradient between car A and car C. (2)

4.3. Find the equation of the tangent to the track at the starting flag. (2)

4.4. At which point between A and B does a car stop turning to the right and start turning to the left? (3)
[13]

QUESTION 5: 15 minutes

The cross section of a hilly region can be drawn as the graph of $y = x^3 - 8x^2 + 16x$, for $0 \le x \le 6$, where *x* is measured in kilometres and *y* is the height above sea level in meters.

5.1. Draw the cross section for $0 \le x \le 6$. Show all calculations. (7)

5.2. Mark these the peak and the valley on your cross section and calculate the difference in height between the two. (2)

5.3. Determine the gradient of the hill at the point P where x = 1. (2)

5.4. Hence, determine the equation of the tangent to the hill at P. (4)
[15]

SECTION B: SOLUTIONS AND HINTS TO SECTION A

QUESTION 1

1.1	$V = \pi r^2 h$ $200 = \pi r^2 h$ $h = \dfrac{200}{\pi r^2}$	✓ $V = \pi r^2 h$ ✓ $200 = \pi r^2 h$ (2)
1.2	$\text{Surface Area} = 2\pi rh + \pi r^2$ $S(r) = \pi r^2 + \dfrac{200}{\pi r^2}.2\pi r$ $S(r) = \pi r^2 + \dfrac{400}{r}$	✓ $S = 2\pi rh + \pi r^2$ ✓ $S(r) = \pi r^2 + \dfrac{200}{\pi r^2}.2\pi r$ (2)

1.3	$S(r)=\pi r^2+400r^{-1}$ $\frac{dS}{dr}=2\pi r-400r^{-2}$ At minimum : $\frac{dS}{dr}=0$ $2\pi r-\frac{400}{r^2}=0$ $\pi r^3-200=0$ $r^3=\frac{200}{\pi}$ $r=3{,}99$ cm	✓ $S(r)=\pi r^2+400r^{-1}$ ✓ $\frac{dS}{dr}=2\pi r-400r^{-2}$ ✓ $2\pi r-\frac{400}{r^2}=0$ ✓ $r^3=\frac{200}{\pi}$ ✓ $r=3{,}99$ cm (5)

[9]

QUESTION 2

2.1.	$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{-3x^2-6xh-3h^2+3x^2}{h}$ $f'(x)=\lim_{h\to 0}\frac{h(-6x-3h)}{h}$ $f'(x)=\lim_{h\to 0}-6x-3h$ $=-6x$ $f(x)=-3x^2$ $f(x+h)=-3(x+h)^2$ $=-3x^2-6xh-3h^2$	✓$-3x^2-6xh-3h^2$ ✓$+3x^2$ ✓ $h(-6x-3h)$ ✓$-6x-3h$ ✓$-6x$ (5)
2.2.	$f(x)=(3x^2-x)^2$ $=9x^4-6x^3+x^2$ $f'(x)=36x^3-18x^2+2x$	✓$9x^4-6x^3+x^2$ ✓$36x^3$ ✓$-18x^2$ ✓$+2x$ (4)
2.3.	$y=\frac{2x^2+3x-2}{x+2}$ $=\frac{(2x-1)(x+2)}{x+2}$ $=2x-1$ $\frac{dy}{dx}=2$	✓$(2x-1)(x+2)$ ✓$2x-1$ ✓$\frac{dy}{dx}=2$ (3)
2.4.	$\frac{d}{dx}\left(\frac{4x^3-3x^2}{2\sqrt{x}}\right)$ $=\frac{d}{dx}\left(\frac{4x^3}{2x^{\frac{1}{2}}}-\frac{3x^2}{2x^{\frac{1}{2}}}\right)$ $=\frac{d}{dx}\left(2x^{\frac{5}{2}}-\frac{3}{2}x^{\frac{3}{2}}\right)$ $=5x^{\frac{3}{2}}-\frac{9}{4}x^{\frac{1}{2}}$	✓$2x^{\frac{5}{2}}$ ✓$-\frac{3}{2}x^{\frac{3}{2}}$ ✓$5x^{\frac{3}{2}}$ ✓$-\frac{9}{4}x^{\frac{1}{2}}$ (4)

[16]

QUESTION 3

3.1.	$\lim_{p\to 4}\frac{p^2-5p+4}{4-p}$ $=\lim_{p\to 4}\frac{(p-4)(p-1)}{-(p-4)}$ $=\lim_{p\to 4}-(p-1)$ $=-(4-1)$ $=-3$	✓$(p-4)(p-1)$ ✓$-(p-4)$ ✓-3 (3)
3.2.	$f(x)=4x^2-4x+\frac{1}{x^2}-\frac{1}{2}$ $f(x)=4x^2-4x+x^{-2}-\frac{1}{2}$ $f'(x)=8x-4-\frac{2}{x^3}$	✓$8x$ ✓-4 ✓$-\frac{2}{x^3}$ (3)
3.3.	$D_x\left[\frac{5x^2-3x}{\sqrt{x}}\right]$ $=D_x\left[5x^{\frac{3}{2}}-3x^{\frac{1}{2}}\right]$ $=\frac{15\sqrt{x}}{2}-\frac{3}{2\sqrt{x}}$	✓$5x^{\frac{3}{2}}$ ✓$-3x^{\frac{1}{2}}$ ✓$\frac{15\sqrt{x}}{2}$ ✓$-\frac{3}{2\sqrt{x}}$ (4)

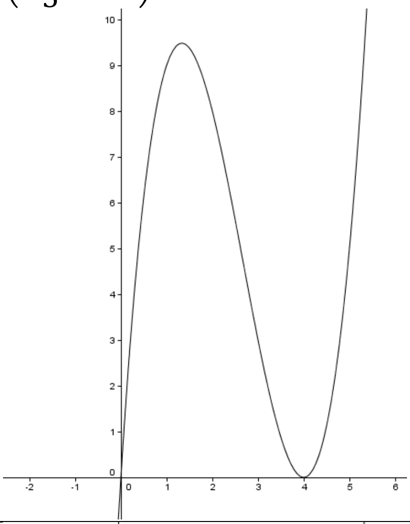
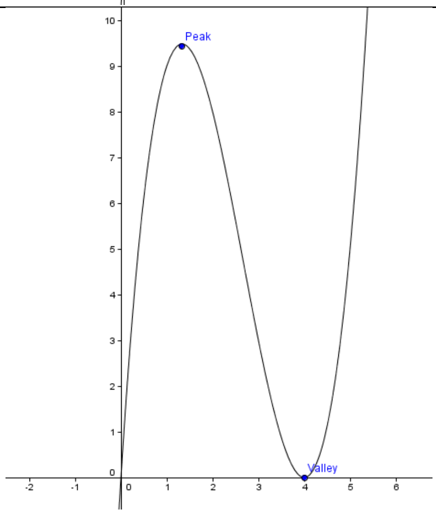
[10]

QUESTION 4

4.1.	$f(x)=\frac{1}{900}x^3-\frac{1}{5}x^2+9x$ $f'(x)=\frac{1}{300}x^2-\frac{2}{5}x+9$ $0=\frac{1}{300}x^2-\frac{2}{5}x+9$ $0=x^2-120+2700$ $0=(x-30)(x-90)$ $x=30 \quad or \quad x=90$ $\therefore A(30;12) \quad B(90;0)$	✓$f'(x)=\frac{1}{300}x^2-\frac{2}{5}x+9$ ✓$0=\frac{1}{300}x^2-\frac{2}{5}x+9$ ✓$0=x^2-120+2700$ ✓$0=(x-30)(x-90)$ ✓ $A(30;12)$ ✓$B(90;0)$ (6)
4.2.	$C(150;600)$ $m_{AC}=\frac{600-120}{150-30}$ $m_{AC}=4$	✓$m_{AC}=\frac{600-120}{150-30}$ ✓$m_{AC}=4$ (2)
4.3.	$f'(x)=\frac{1}{300}(0)^2-\frac{2}{5}(0)+9$ $m=9$ $\therefore y=9x$	✓$m=9$ ✓$\therefore y=9x$ (2)
4.4.	$f''(x)=\frac{1}{150}x-\frac{2}{5}$ $0=\frac{1}{150}x-\frac{2}{5}$ $0=x-60$ $x=60$ $\therefore (60;60)$	✓$\frac{1}{150}x-\frac{2}{5}$ ✓$0=\frac{1}{150}x-\frac{2}{5}$ ✓$(60;60)$ (3)

[13]

QUESTION 5

5.1.	$y = x^3 - 8x^2 + 16x$ $0 = x(x^2 - 8x + 16)$ $0 = x(x-4)(x-4)$ $x = 0 \quad x = 4$ $(0{,}0)(4;0)$ $3x^2 - 16x + 16 = 0$ $(3x-4)(x-4) = 0$ $x = \frac{4}{3} \qquad x = 4$ $y = 9{,}48 \quad y = 0$ $\left(1\frac{1}{3}; 9{,}48\right)$ $6x - 16 = 0$ $6x = 16$ $x = 2\frac{2}{3}$ $y = 4{,}74$ $\left(2\frac{2}{3}; 4{,}74\right)$ 	*x*-intercepts: ✓$(0{,}0)$ ✓$(4;0)$ Turning points: ✓$(4;0)$ ✓$\left(1\frac{1}{3}; 9{,}48\right)$ Inflection: ✓$\left(2\frac{2}{3}; 4{,}74\right)$ ✓✓plotted function (7)
5.2.	 Difference in height = 9,48km	✓marking the valley and peak on function ✓9,48km (2)

5.3.	$m = 3x^2 - 16 + 16$ $m = 3(1)^2 - 16(1) + 16$ $m = 3$	✓$m = 3x^2 - 16 + 16$ ✓$m = 3$ (2)
5.4.	$y = (1)^3 - 8(1)^2 + 16 = 9$ $(1;9) \quad m = 3$ $\therefore y - 9 = 3(x - 1)$ $y = 3x + 6$	✓$(1;9)$ ✓$y - 9 = 3(x - 1)$ ✓$3x$ ✓$+6$ (4) **[15]**

SECTIONC: HOMEWORK

QUESTION 1

A builder wishes to construct a steel window frame in the shape of a rectangle with a semi-circular part on top. The radius of the semi-circular part is r metres and the width of the rectangular part is h metres.

1.1 Write down, in terms of $h$and r

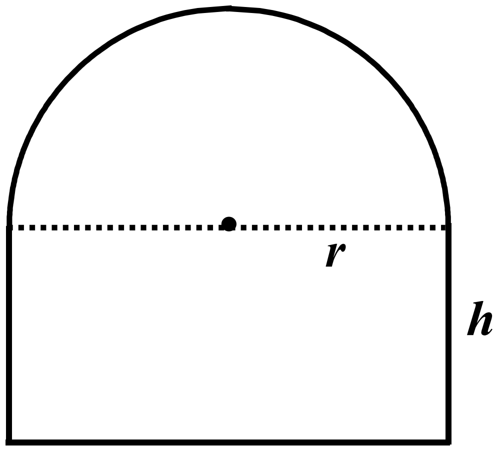
1.1.1 the steel perimeter (P) of the frame. (2)

1.1.2 the area enclosed by the frame. (2)

1.2 The area enclosed by the frame is to be 4 square metres.
Show that the perimeter (P) is

$$P = \left(\frac{\pi}{2} + 2\right)r + \frac{4}{r}$$ (4)

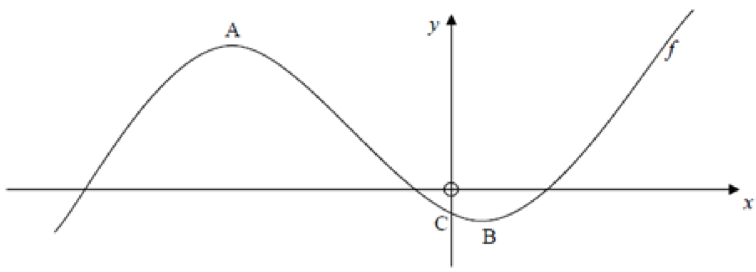
1.3 If the steel for the frame costs R10 per metre, calculate the value of r for which the total cost of the steel will be a minimum. (4)



[12]

QUESTION 2

Refer to the figure. The graph (not drawn to scale) of $f(x) = 4x^3 + 27x^2 - 30x - 1$ is shown with A and B the turning points of the graph.



2.1. Determine the coordinates of A and B. (6)

2.2. Calculate the average gradient of *f* between the points A and B. (2)

2.3. C is the *y*-intercept of the graph. Determine the equation of the tangents to *f* at C. (3)

2.4. Determine the *x*-coordinate of the point on *f* where this tangent cuts the graph again. (3)

[14]

SECTION D: SOLUTIONS TO HOMEWORK

QUESTION 1

1.1.1	$P = 2h + 2r + \frac{1}{2} \times 2\pi r$ $\therefore P = 2h + 2r + \pi r$	✓ $2h + 2r$ ✓ πr (2)
1.1.2	$A = 2rh + \frac{1}{2}\pi r^2$	✓ $2rh$ ✓ $\frac{1}{2}\pi r^2$ (2)
1.2	$4 = 2rh + \frac{1}{2}\pi r^2$ $\therefore 8 = 4rh + \pi r^2$ $\therefore 8 - \pi r^2 = 4rh$ $\therefore \frac{8 - \pi r^2}{4r} = h$ $P = 2h + 2r + \pi r$ $\therefore P = 2\left(\frac{8 - \pi r^2}{4r}\right) + 2r + \pi r$ $\therefore P = \frac{8 - \pi r^2}{2r} + 2r + \pi r$ $\therefore P = \frac{4}{r} - \frac{\pi r}{2} + 2r + \pi r$ $\therefore P = \frac{4}{r} + \frac{\pi r}{2} + 2r$ $\therefore P = \frac{4}{r} + \left(\frac{\pi}{2} + 2\right)r$ $\therefore P = \left(\frac{\pi}{2} + 2\right)r + \frac{4}{r}$	✓ $4 = 2rh + \frac{1}{2}\pi r^2$ ✓ $\frac{8 - \pi r^2}{4r} = h$ ✓ $P = 2\left(\frac{8 - \pi r^2}{4r}\right) + 2r + \pi r$ ✓ $P = \left(\frac{\pi}{2} + 2\right)r + \frac{4}{r}$ (4)

1.3	$C=10\left(\frac{\pi}{2}+2\right)r+\frac{40}{r}$ $\therefore C=5\pi r+20r+40r^{-1}$ $\therefore C'(r)=5\pi+20-40r^{-2}$ $\therefore C'(r)=5\pi+20-\frac{40}{r^2}$ $\therefore 0=5\pi+20-\frac{40}{r^2}$ $\therefore \frac{40}{r^2}=5\pi+20$ $\therefore \frac{40}{5\pi+20}=r^2$ $\therefore \sqrt{\frac{40}{5\pi+20}}=r$ $\therefore r=1{,}06\text{m}$	✓ $C=10\left(\frac{\pi}{2}+2\right)r+\frac{40}{r}$ ✓ $C=5\pi r+20r+40r^{-1}$ ✓ $0=5\pi+20-\frac{40}{r^2}$ ✓ $r=1{,}06\text{m}$ (4) **[12]**

QUESTION 2

2.1.	$At\ A\ and\ B:\ \ f'(x)=0$ $f'(x)=12x^2+54x-30=0$ $2x^2+9x-5=0$ $(2x-1)(x+5)=0$ $x=\frac{1}{2}\quad or\quad x=-5$ $f\left(\frac{1}{2}\right)=4\left(\frac{1}{2}\right)^3+27\left(\frac{1}{2}\right)^2-30\left(\frac{1}{2}\right)-1$ $=\frac{-35}{4}(-8{,}75)$ $f(-5)=4(-5)^3+27(-5)^2-30(-5)-1$ $=324$ $\therefore A(-5;324),\qquad B\left(\frac{1}{2};\frac{-35}{4}\right)$	✓$f'(x)=0$ ✓substitution of x values ✓✓$A(-5;324)$ ✓✓$B\left(\frac{1}{2};\frac{-35}{4}\right)$ (6)
2.2.	$Ave\ Grad=\frac{324-\left(\frac{-35}{4}\right)}{-5-\frac{1}{2}}$ $=\frac{-121}{2}\quad(-60{,}5)$	✓subs x and y values ✓$=\frac{-121}{2}\quad(-60{,}5)$ (2)
2.3.	$C(0;-1)$ $f'(0)=-30$ $Equ.of\ tangent: y=-30x-1$	✓ $C(0;-1)$ ✓$f'(0)=-30$ ✓ $y=-30x-1$(3)
2.4.	$4x^3+27x^2-30x-1=-30x-1$ $4x^3+27x^2=0$ $x^2(4x+27)=0$ $x=0\quad or\quad x=-\frac{27}{4}$ $\therefore x=\frac{-27}{4}$	✓cubic=tangent ✓$x^2(4x+27)=0$ ✓ $x=\frac{-27}{4}$ (3)

[14]

SESSION 18.1

TOPIC: LINEAR PROGRAMMING

Teacher Note: This session covers the basic rules and methods for solving Linear Programming questions. An illustration of the different kinds of questions that may be asked, is given and the theory is addressed. There are 2 solved questions and 1 additional homework question.

LESSON OVERVIEW

1.	Introduction to Linear Programming:	5 minutes
2.	Typical exam questions:	
	Question 1:	30 minutes
	Question 2:	30 minutes
3.	Homework Question:	25 minutes

SECTION A: TYPICAL EXAM QUESTIONS

QUESTION 1

Magical Homes, a home decor company, must produce at least 90 bedroom lamps per week. Not more than 18 people can be employed. An artisan, who earns R600 per week, can produce 9 lamps per week, while an apprentice, who earns R300 per week, only 5 lamps per week. At least one apprentice must be employed for every five artisans. The ratio of apprentices to artisans must not exceed 1:2. Assume that the number of artisans is x and the number of apprentices is y.

1.1 Write down the inequalities. (4)

1.2 Sketch the feasible region. (5)

1.3 How many artisans and apprentices should be employed in order to minimise the wages paid? (7)

[16]

QUESTION 2

TV Mania is a company manufacturing television sets. The company produces two types of television sets: TX and TY. At least one set of both must be manufactured per week. At most 6 sets of TX and 5 sets of TY can be manufactured per week. It takes five hours to manufacture a TX set and 4 hours to manufacture a TY set. A working week is 40 hours. Twice as many workers are required to manufacture TX sets than TY sets, while at least 8 workers are constantly occupied in the manufacturing process.

1.1 Write down the constraints if the company produces x TX sets and y TY sets. (5)

1.2 Represent the system of constraints on graph paper. Indicate the whole number points in the feasible region. (5)

1.3 The company makes a profit of R500 on TX and R250 on TY.

1.3.1 Determine the number of each television set to be manufactured weekly if the company wants to maximise profits. (2)

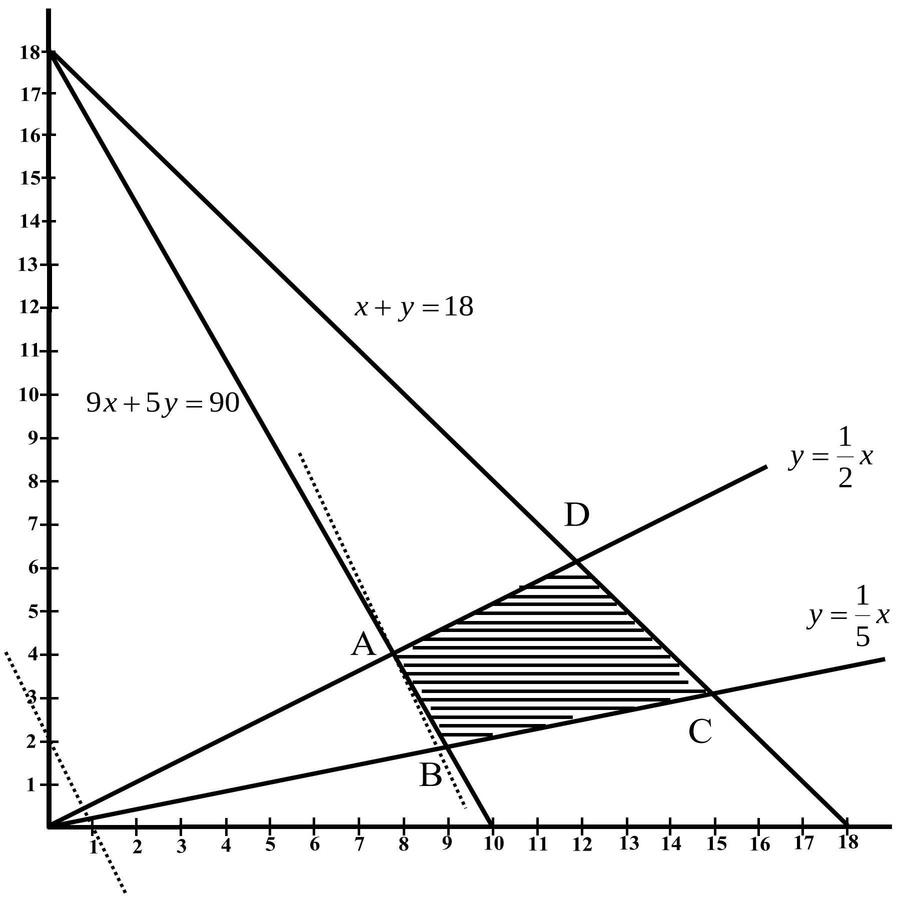
1.3.2 What number of each set will yield a minimum profit per week? (2)

[14]

SECTION B: SOLUTIONS AND HINTS TO SECTION A

QUESTION 1

1.1	Let the number of artisans be x and the number of apprentices be y. $x+y\leq 18$ $9x+5y\geq 90$ $y\leq \frac{1}{2}x$ $y\geq \frac{1}{5}x$	✓ $x+y\leq 18$ ✓ $9x+5y\geq 90$ ✓ $y\leq \frac{1}{2}x$ ✓ $y\geq \frac{1}{5}x$ (4)
1.2	See diagram below	✓ $x+y\leq 18$ ✓ $9x+5y\geq 90$ ✓ $y\leq \frac{1}{2}x$ ✓ $y\geq \frac{1}{5}x$ ✓ Feasible region (5)



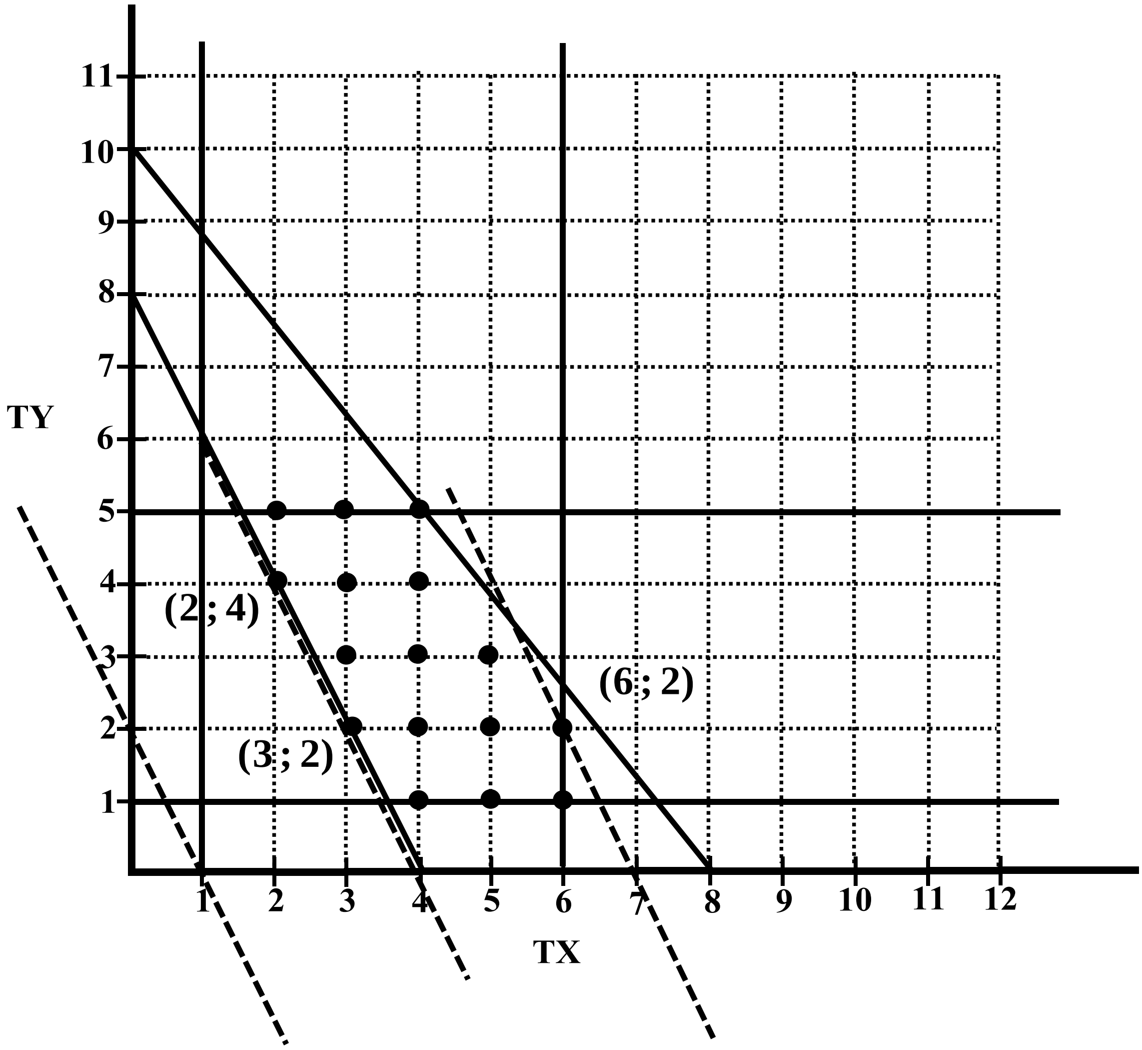
1.3	$C = 600x + 300y$ $\therefore 600x + 300y = C$ $\therefore 300y = -600x + C$ $\therefore y = -2x + \frac{C}{300}$ Point A $\left(7\frac{19}{23}; 3\frac{21}{23}\right)$ produces minimum cost. Therefore, the company will need to employ: 8 artisans and 4 apprentices	✓ $C = 600x + 300y$ ✓ $y = -2x + \frac{C}{300}$ ✓✓✓✓ $\left(7\frac{19}{23}; 3\frac{21}{23}\right)$ Note: Simultaneous equations must be used to obtain the value of *x* an *y* ✓ 8 and 4 (7)

[16]

QUESTION 2

2.1	$1 \le x \le 6$ $1 \le y \le 5$ $5x + 4y \le 40$ $2x + y \ge 8$ $x ; y \in N$	✓ $1 \le x \le 6$ ✓ $1 \le y \le 5$ ✓ $5x + 4y \le 40$ ✓ $2x + y \ge 8$ ✓ $x ; y \in N$ (5)

2.2	see diagram below	✓ $1 \le x \le 6$ ✓ $1 \le y \le 5$ ✓ $5x + 4y \le 40$ ✓ $2x + y \ge 8$ ✓ $x ; y \in N$ (5)
2.3.1	$500x + 250y = P$ $\therefore 250y = -500x + P$ $\therefore y = -2x + \frac{P}{250}$ Max at $(6 ; 2)$	✓ $500x + 250y = P$ ✓ $(6 ; 2)$ (2)
2.3.2	Min at $(2 ; 4)$ and $(3 ; 2)$	✓ $(2 ; 4)$ ✓ $(3 ; 2)$ (2)



[14]

SECTION C: HOMEWORK

QUESTION 1

A shopkeeper intends buying up to 25 second-hand radios. He has a choice between two types, one without FM for R 30 each and one with FM costing R 40 each.

His total expenditure on the radios must not exceed R 840. If he can sell the first type at a profit of R 10 each and the second type at a profit of R 12 each, determine

1.1 The set of inequalities (3)

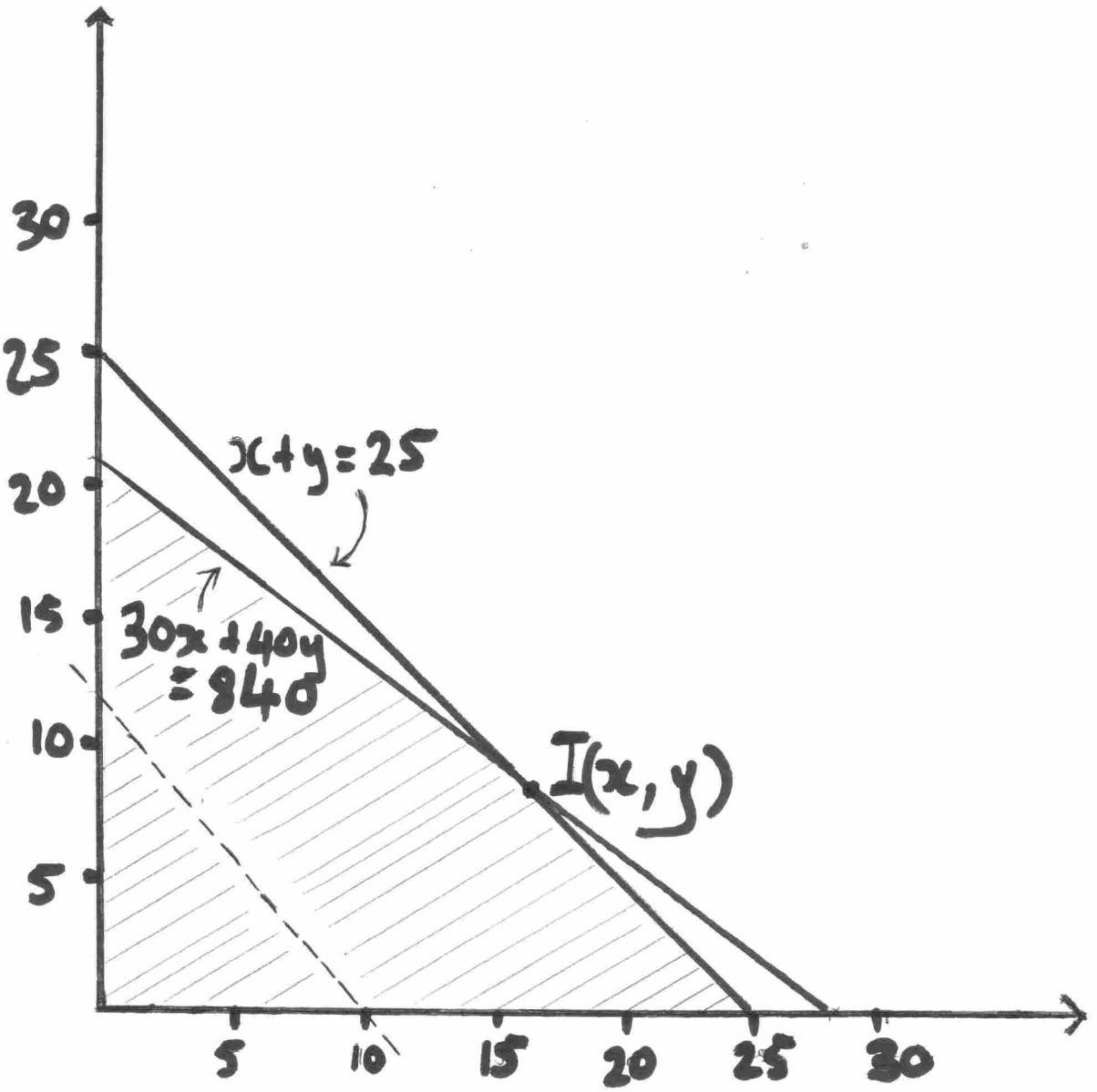
1.2 Sketch the graph and feasible region (5)

1.3 How many of each he should purchase to ensure a maximum profit. (4)

[12]

SECTION D: HOMEWORK SOLUTIONS

1.1	$x + y \leq 25$ $30x + 40y \leq 840$ $x \leq 0$ $y \leq 0$ $x, y \in \mathbb{N}$	✓ $x + y \leq 25$ ✓ $30x + 40y \leq 840$ ✓ $x \leq 0$, $y \leq 0$, $x, y \in \mathbb{N}$ (3)
1.2	see diagram on next page	✓ $x + y \leq 25$ ✓ $30x + 40y \leq 840$ ✓ $x \leq 0$ ✓ $y \leq 0$ ✓ $x, y \in \mathbb{N}$ (5)
1.3	$10x + 12y = P$ $\therefore y = -1.2x + \frac{P}{12}$ Intersection $I(x, y)$of $x + y = 25$ and $30x + 40y = 840$ $\therefore I(x, y) = (16, 9)$ Max at either $I(x, y)$or $(25, 0)$ Max at $I(x, y), P = 268$ $\therefore x = 16, y = 9$	✓ $10x + 12y = P$ ✓ Intersection $I(x, y)$of $x + y = 25$ and $30x + 40y = 840$ ✓ Check P at $I(x, y)$ and $(25, 0)$ ✓ Max at $I(x, y), P = 268$ (4)



[12]

SESSION 18.2

TOPIC: TRIGONOMETRY

Teacher Note: Learners need to know all of the previous Trigonometry they have studied as it will be examined as part of their knowledge in the final exam.
If specifically asked to simplify without a calculator, marks will not be awarded should calculator usage be apparent. The trig identities from grade 11 will not be given and must be learnt
i.e. $\sin^2 x + \cos^2 x = 1$ *and* $tan\theta = \frac{sin\theta}{cos\theta}$

SECTION A: TYPICAL EXAM QUESTIONS

QUESTION 1

1.1 Simplify:

$$\frac{\cos(360+x).\cos(90+x).\tan^2(x-180)}{\sin(180-x).\sin(x-360).\tan(-x)}$$ [5]

QUESTION 2

If sin12˚ = k determine the following in terms of k:

2.1 sin192˚ (2)

2.2 cos258˚ (3)

2.3 sin336˚ (5)

[10]

QUESTION 3

3.1 Prove that: $\frac{sin2A+cos2A+1}{sinA+cosA} = 2cosA$ (6)

3.2 For which values of A is the above identity not defined, for A Є [-180; 180] (3)

[9]

QUESTION 4

Prove without the use of a calculator, that if sin28° = a and cos32° = b, then:

$$b\sqrt{1-a^2} - a\sqrt{1-b^2} = \frac{1}{2}$$

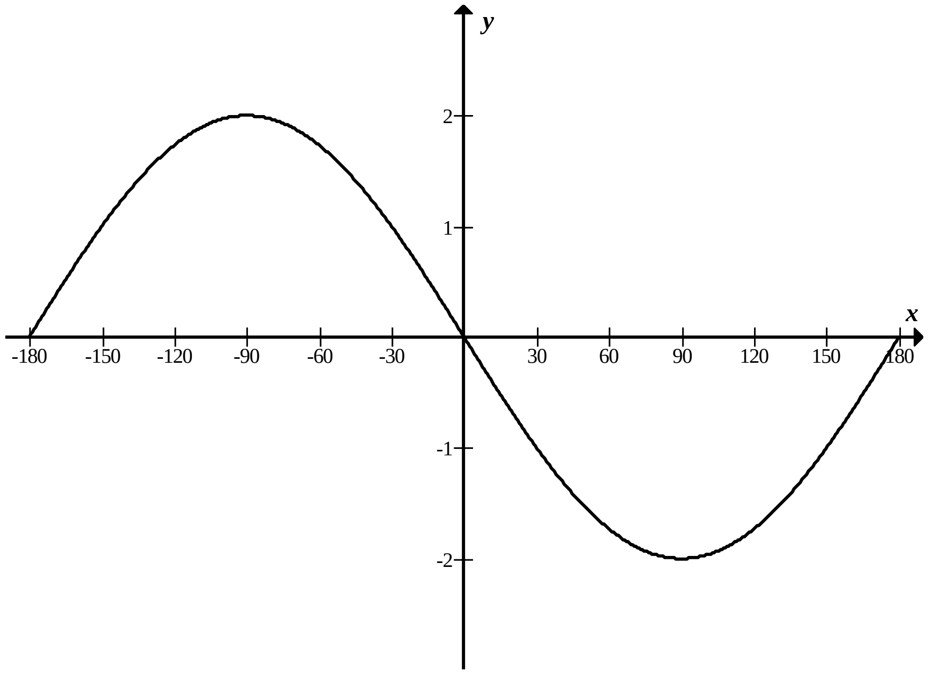
[4]

QUESTION 5

Determine the general solution of: $\sin^2 x + \cos 2x - \cos x = 0$ [7]

QUESTION 6

The graph of f(x)=-2sinx is drawn below



6.1 Write down the period of f. (1)

6.2 Write down the amplitude of h if $h(x) = \frac{f(x)}{4}$ (2)

6.3 Draw the graph of $g(x) = \cos(x - 30°)$ for $x \in [-180° ; 180°]$ (3)

6.4 Use the graph to determine the number of solutions for $-2\sin x = \cos(x - 30°)$, $x \in [-180° ; 180°]$ (1)

6.5 For which values of x is g(x) ≥0? (2)

6.6 For which values of x is $f'(x) < 0$ and $g'(x) > 0$? (3)

[12]

SECTION B: SOLUTIONS TO SECTION A

QUESTION 1

$$\frac{\cos(360+x).\cos(90+x).\tan^2(x-180)}{\sin(180-x).\sin(x-360).\tan(-x)} = \frac{\cos x\,(-\sin x).\tan^2 x}{\sin x.\sin x.(-\tan x)}$$

$$= \frac{\cos x.\tan x}{\sin x}$$

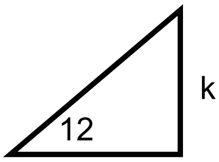
$$= \frac{\cos x.\sin x}{\sin x.\cos x} = 1$$ **[6]**

QUESTION 2

2.1 sin192 = sin(180+12) = -sin12 = -k (2)

2.2 cos258 = cos(180+78) = -cos78
= -cos(90-12) = -sin12 = -k (3)

2.3 sin336 = sin(360-24)
= -sin24 = -sin(2$\times$ 12) = -2sin12.cos12
= -2k($\sqrt{(1-k^2})$) (5)



[10]

QUESTION 3

3.1 $$\frac{sin2A+cos2A+1}{sinA+cosA} = \frac{2sinAcosA+2cos^2A-1+1}{sinA+cosA}$$

$$= \frac{2cosA(sinA+cosA)}{(sinA+cosA)} = 2cosA$$ (6)

3.2 Identity is undefined if sinA +cosA = 0
sinA = -cosA
tanA = -1 cosA≠0
undefined when A = 135 +k.180 k$\in\mathbb{Z}$ (3)

[9]

QUESTION 4

$$b\sqrt{1-a^2} - a\sqrt{1-b^2}$$
$$= \cos 32°.\sqrt{1-\sin^2 28°} - \sin 28°\sqrt{1-\cos^2 32°}$$
$$= \cos 32°.\cos 28° - \sin 28°.\sin 32°$$
$$= \cos(32° + 28°)$$
$$= \cos 60°$$
$$= \frac{1}{2}$$ **[4]**

QUESTION 5

$$\sin^2 x + \cos 2x - \cos x = 0$$
$$\sin^2 x + (\cos^2 x - \sin^2 x) - \cos x = 0$$
$$\cos^2 x - \cos x = 0$$
$$\cos x(\cos x - 1) = 0$$
$$\cos x = 0 \ \ or \ \cos x = 1$$
$$x = \pm 90° + k.360° \quad or \ x = 0° + k.360° \quad k \in Z$$
$$= k.360°$$

$$(\text{i.e. } x = 90° + k.180° \ \text{ or } \ x = k.360° \pm 90°, k \in Z)$$

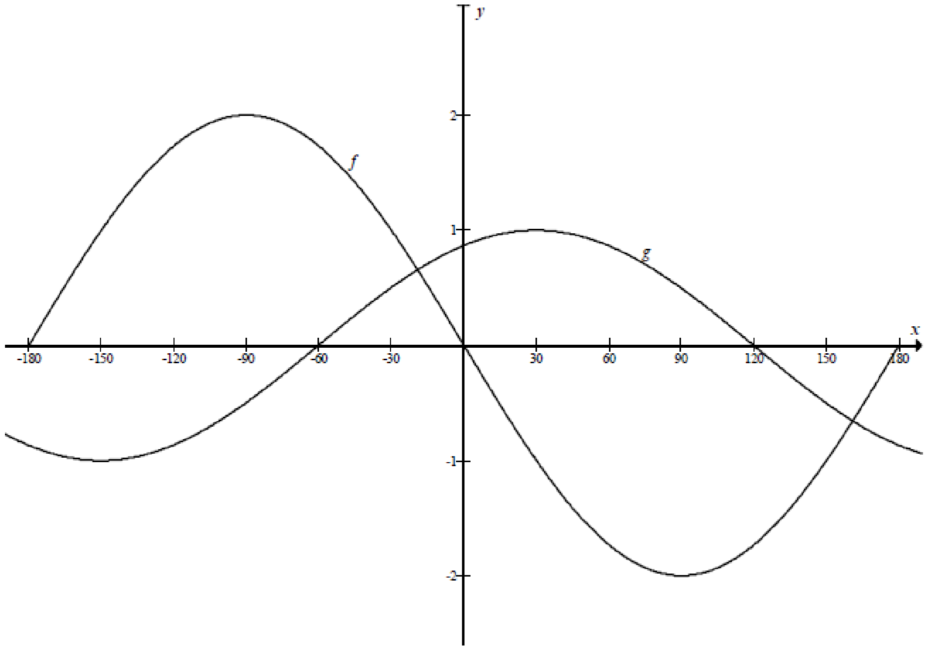
[7]

QUESTION 6

6.1 Period = 360˚ (1)

6.2 Amplitude = ½ (2)

6.3 (3)



6.4 2 solutions (1)

6.5 -60˚≤ x ≤ 120˚ or x€[-60˚; 120˚] (2)

6.6 -90˚< x < 30˚ or x€(-90˚; 30˚) (3)

[12]

SECTION B: HOMEWORK

QUESTION 1

Determine the value of the following without using a calculator:

$$\frac{\sin(-145°).\cos(-215°)}{\sin 510°.\cos 340°}$$ [8]

QUESTION 2

If $\sin 2A = \frac{\sqrt{5}}{3}$, with $2A \in [90° : 270°]$ determine without the use of a calculator:

2.1 cos2A (4)

2.2 tan2A (1)

2.3 sinA (4)

[9]

QUESTION 3

Simplify completely: $\frac{\sin(90+\theta)+\cos(180+\theta)\sin(-\theta)}{sin180-tan135}$ [5]

QUESTION 4

Prove that for any angle A:

$$\frac{4sinA.cosA.cos2A.sin15}{sin2A(tan225-2\sin^2 A} = \frac{\sqrt{6}-\sqrt{2}}{2}$$ [6]

QUESTION 5

Determine the general solution of:

$6cosx - 5 = \frac{4}{cosx}$; $cosx \neq 0$ [6]

QUESTION 6

Determine the value of the following without using a calculator:

$\cos^4 375° - \sin^4 345°$ [6]

QUESTION 7

If $\sin 19° = t$, express the following in terms of *t*. (Make use of a sketch)

7.1 sin79° (7)

7.2 tan71° (3)

[10]

SECTION D: HOMEWORK SOLUTIONS

QUESTION 1

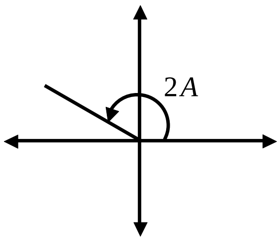
$$\frac{\sin(-145°).\cos(-215°)}{\sin 510°.\cos 340°}$$
$$=\frac{(-\sin 145°)(\cos 215°)}{(\sin 150°)(\cos 20°)}$$
$$=\frac{(-\sin 35°)(-\cos 35°)}{(\sin 30°)(\cos 20°)}$$

$$=\frac{\sin 35 \cos 35°°}{\left(\frac{1}{2}\right)(\cos 20°)}$$
$$=\frac{2\sin 35 \cos 35°°}{\cos 20°}$$
$$=\frac{\sin 70°}{\cos 20°}$$
$$=\frac{\cos 20°}{\cos 20°}$$
$$=1$$

[8]

QUESTION 2

$$\sin 2A = \frac{\sqrt{5}}{3}$$
$$x^2 = r^2 - y^2$$
$$x^2 = 3^2 - \left(\sqrt{5}\right)^2$$
$$x^2 = 4$$
$$x = \pm 2$$
$$\therefore x = -2$$
$$\cos 2A = \frac{-2}{3}$$



[9]

QUESTION 3

$$\frac{\sin(90° + \theta) + \cos(180° + \theta)\sin(-\theta)}{\sin 180° - \tan 135°}$$
$$=\frac{\cos\theta + (-\cos\theta)(-\sin\theta)}{0+1}$$
$$=\cos\theta + \cos\theta.\sin\theta$$
$$=\cos\theta(1+\sin\theta)$$

[5]

QUESTION 4

$$\frac{4\sin A\cos A\cos 2A.\sin 15^\circ}{\sin 2A(1-2\sin^2 A)}$$
$$=\frac{4\sin A\cos A\cos 2A.\sin 15^\circ}{2\sin A\cos A(1-2\sin^2 A)}$$
$$=\frac{2\cos 2A.\sin 15^\circ}{\cos 2A}$$
$$=2\sin 15^\circ$$
$$=2\sin(45^\circ-30^\circ)$$
$$=2[\sin 45^\circ\cos 30^\circ-\cos 45^\circ\sin 30^\circ]$$
$$=2\left[\frac{\sqrt{2}}{2}.\frac{\sqrt{3}}{2}-\frac{\sqrt{2}}{2}.\frac{1}{2}\right]$$
$$=2\left[\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}\right]$$
$$=\frac{\sqrt{6}-\sqrt{2}}{2}$$

[6]

QUESTION 5

$$6\cos x-5=\frac{4}{\cos x}$$
$$6\cos^2 x-5\cos x=4$$
$$6\cos^2 x-5\cos x-4=0$$
$$(3\cos x-4)(2\cos x+1)=0$$
$$\cos x=\frac{4}{3} \quad or \quad \cos x=\frac{-1}{2}$$
no solution or $x=120^\circ+k.360^\circ\ ,k\in Z$

or

$x=240^\circ+k.360^\circ\ ,k\in Z$

Alternative solution for $\cos x=\frac{-1}{2}$

$x=k.360^\circ\pm 120^\circ\ k\in Z$

> **Note:**
> If candidate puts $\pm k.360$ then $k\in \mathrm{N}_0$

[6]

QUESTION 6

$$\cos^4 375° - \sin^4 345°$$
$$= \cos^4 15° - \sin^4 15°$$
$$= (\cos^2 15° + \sin^2 15°)(\cos^2 15° - \sin^2 15°)$$
$$= (1)(\cos 30°)$$
$$= \frac{\sqrt{3}}{2}$$

[6]

QUESTION 7

7.1

$$\sin 19° = \frac{t}{1}$$
$$x^2 + t^2 = 1^2$$
$$\therefore x^2 = 1 - t^2$$
$$\therefore x = \sqrt{1-t^2}$$
$$\sin 79°$$
$$= \sin(19° + 60°)$$
$$= \sin 19° \cos 60° + \cos 19° \sin 60°$$
$$= (t)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{1-t^2}}{1}\right)\left(\frac{\sqrt{3}}{2}\right)$$
$$= \frac{t + \sqrt{3}\sqrt{1-t^2}}{2} = \frac{t + \sqrt{3-3t^2}}{2}$$

(7)

7.2

$$\tan 71°$$
$$= \frac{\sin 71°}{\cos 71°}$$
$$= \frac{\cos 19°}{\sin 19°}$$
$$= \frac{\frac{\sqrt{1-t^2}}{1}}{t}$$
$$= \frac{\sqrt{1-t^2}}{t}$$

(3)
[10]

SESSION 19.1

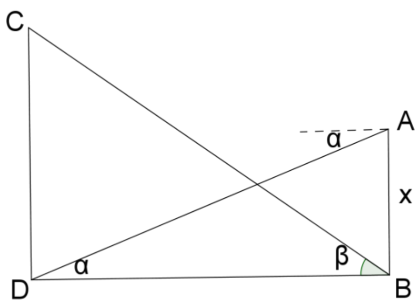
TOPIC: 2D TRIGONOMETRY

Teacher Note: Learners must revise the sine, cos and area rules to help them 'solve' triangles.

SECTION A: TYPICAL EXAM QUESTIONS

QUESTION 1

AB and CD are two towers in the same horizontal plane, the angle of depression of D from A is α and the angle of elevation of C from B is β.



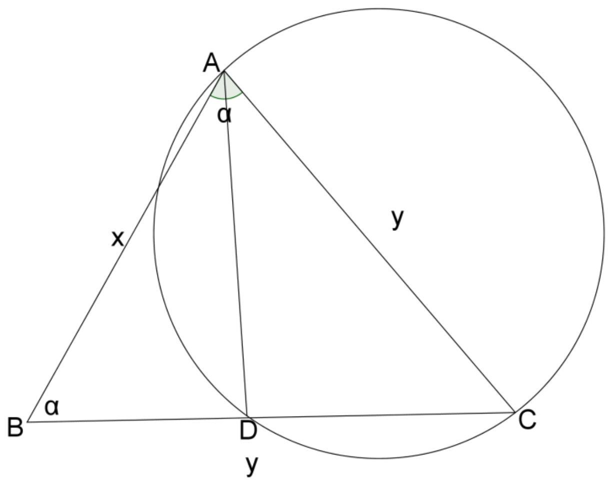
1.1 If AB =x prove that CD = $\frac{x\tan\beta}{\tan\alpha}$ (4)

1.2 Hence find CD without a calculator when x = $50\sqrt{3}$, β = 45 and α = 30 (2)

[6]

QUESTION 2

In the diagram below; AC is a diameter of the circle with AB = x , AC = BC = y and ∠ABC = α and ∠ADC = 90



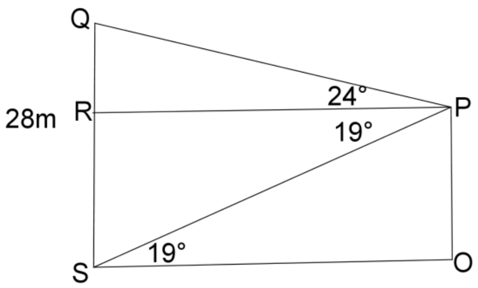
a) Show that cosα = $\frac{}{2y}$ (3)

b) Determine DC in terms of x and y (4)

[7]

QUESTION 3

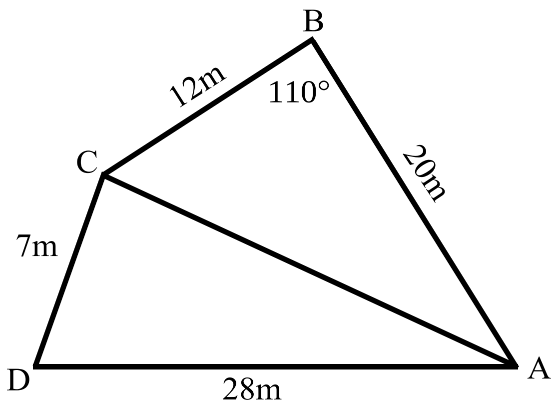
From a point P, on top of a building the angle of elevation to the top of a cell phone tower is 24° and the angle of depression to the foot of the tower is 19°, as shown below:



If the height of the tower is 28m how far is the building from the tower, if they lie in the same horizontal plane? (Round off to the nearest whole number). [8]

QUESTION 4

A piece of land has the form of a quadrilateral ABCD with $AB = 20\text{m}$, $BC = 12\text{m}$, $CD = 7\text{m}$ and $AD = 28\text{m}$, $\hat{B} = 110^\circ$
The owner decides to divide the land into two plots by erecting a fence from A to C.



(a) Calculate the length of the fence AC correct to one decimal place. (2)

(b) Calculate the size of $B\hat{A}C$ correct to the nearest degree. (2)

(c) Calculate the size of $\hat{D}$, correct to the nearest degree. (3)

(d) Calculate the area of the entire piece of land ABCD, correct to one decimal place. (3)

[10]

QUESTION 5

ΔABC is an isosceles triangle with $AB = BCm$, $AB = c$, $AC = b$ and $BC = a$.

Prove that $\cos B = 1 - \frac{b^2}{2a^2}$ [4]

SECTION B: SOLUTIONS TO SECTION A

QUESTION 1

1.1 Tanα = x/DB
DB = x/tanα

In ΔCBD: tanβ = CD/DB
CD = $DBtan\beta = \frac{xtan\beta}{tan\alpha}$ (4)

1.2 CD = $\frac{50\sqrt{3}tan45}{tan30} = \frac{50\sqrt{3}\left(\frac{1}{\sqrt{2}}\right)}{\frac{1}{\sqrt{3}}}$ = $150\left(\frac{1}{\sqrt{2}}\right) = \frac{150\sqrt{2}}{2} = 75\sqrt{2}$ (2)

[6]

QUESTION 2

(a) $AC^2 = AB^2 + BC^2 - 2AB.BCcos\alpha$
$y^2 = x^2 + y^2 - 2xycos\alpha$
$-2xycos\alpha = -x^2$
$\therefore cos\alpha = \frac{{}^{2}}{2xy} = \frac{}{2y}$ (3)

(b) In ΔABD: cosα = BD/AB
BD = $ABcos\alpha = x\left(\frac{}{2y}\right) = \frac{{}^{2}}{2y}$
$\therefore DC = y - \frac{{}^{2}}{2y} = \frac{2y^2 - x^2}{2y}$ (4)

[7]

QUESTION 3

In ΔPQS: $\angle QSP = 71$ $QS \perp SO$, $\angle Q = 66$ $(angles\ in\ triangle\ PQS)$
$\frac{28}{sin43} = \frac{PS}{sin66}$

$PS = \frac{28sin66}{sin43} = 38m$

In ΔPSO: $cos19 = \frac{OS}{PS}$
$\therefore OS = 38cos19 = 36$ **[8]**

QUESTION 4

(a)

$$\mathrm{AC}^2 = (12m)^2 + (20m)^2 - 2(12m)(20m)\cos 110^\circ$$

$$\therefore \mathrm{AC}^2 = 708{,}1696688$$

$$\therefore \mathrm{AC} = 26{,}6m$$

(3)

b)

$$\frac{\sin \mathrm{B\hat{A}C}}{12m} = \frac{\sin 110^\circ}{26{,}6m}$$

$$\therefore \sin \mathrm{B\hat{A}C} = \frac{12 \times \sin 110^\circ}{26{,}6m}$$

$$\therefore \sin \mathrm{B\hat{A}C} = 0{,}4239214831$$

$$\therefore \mathrm{B\hat{A}C} = 25^\circ$$

(3)

OR

$$(12m)^2 = (20m)^2 + (26{,}6m)^2 - 2(20m)(26{,}6m)\cos \mathrm{B\hat{A}C}$$

$$\therefore 1064 \cos \mathrm{B\hat{A}C} = 963{,}56m^2$$

$$\therefore \cos \mathrm{B\hat{A}C} = 0{,}9056015038$$

$$\therefore \mathrm{B\hat{A}C} = 25^\circ$$

c)

$$(26{,}6m)^2 = (7m)^2 + (28m)^2 - 2(7m)(28m)\cos \hat{\mathrm{D}}$$

$$\therefore 392 \cos \hat{\mathrm{D}} = 125{,}44$$

$$\therefore \cos \hat{\mathrm{D}} = 0{,}32$$

$$\therefore \hat{\mathrm{D}} = 71^\circ$$

(2)

d)

Area ABCD

$$= \frac{1}{2}(12m)(20m)\sin 110^\circ + \frac{1}{2}(7m)(28m)\sin 71^\circ$$

$$= 205{,}4m^2$$

(2)

[10]

QUESTION 5

$$b^2 = a^2 + c^2 - 2ac\cos \mathrm{B}$$

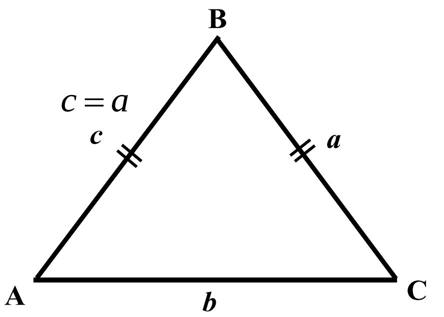
$$\therefore b^2 = a^2 + a^2 - 2a \times a \times \cos \mathrm{B}$$

$$\therefore b^2 = 2a^2 - 2a^2 \cos B$$

$$\therefore b^2 = 2a^2(1 - \cos B)$$

$$\therefore \frac{b^2}{2a^2} = 1 - \cos B$$

$$\therefore \cos \mathrm{B} = 1 - \frac{b^2}{2a^2}$$



[4]

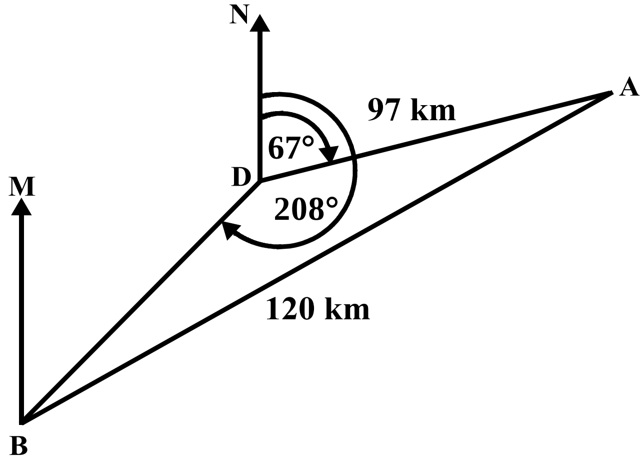
SECTION C: HOMEWORK

QUESTION 1

A mathematician stands on a bank of a river and finds that the angle of elevation to the top of a tree on the opposite bank is 18,3˚. If she moves 45m backwards in line with her first position and the tree, she finds the angle of elevation is now 13,7˚. Calculate the height of the tree and the width of the river (to the nearest metre).
(Tip: A diagram must be used in this question)

QUESTION 2

Two ships, A and B, are 120 km apart. Ship A is at a bearing of 67˚ from D and 97 km away from D. DN points due north. Ship B is at a bearing of 208˚ from D.

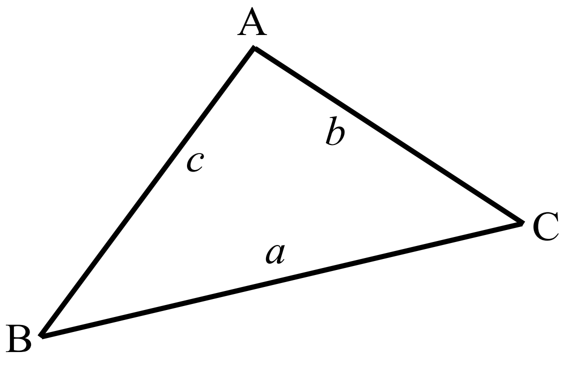


(a) Determine the bearing of Ship A from Ship B ($\angle MBA$), when BM∥DN

(b) If Ship B travels due north, and Ship B travels due south, then at some instant of time, Ship A is due east of Ship B. Calculate the distance between the two ships at that instant.

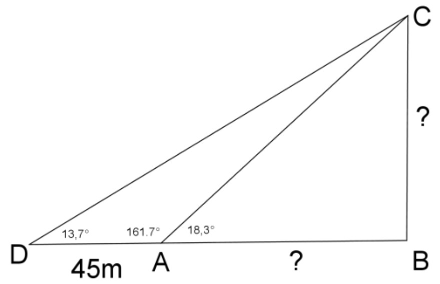
QUESTION 3

If b=c and $a^2 = 7b^2$, show why it is impossible to construct ∆ABC.



SECTION D: SOLUTIONS TO HOMEWORK

QUESTION 1



In ΔCDA $\angle DAC = 180 - 18{,}3 = 161{,}7^{\circ}$
$\angle DCA = 180 - (13{,}7 + 161{,}7) = 4{,}6^{\circ}$

$$\frac{AC}{sin13{,}7} = \frac{45}{sin4{,}6}$$

$$\therefore AC = \frac{45sin13{,}7}{sin4{,}6} = 133m$$

In ΔABC $sin18{,}3 = \frac{BC}{AC} = \frac{BC}{132{,}89}$
$\therefore BC = 132{,}89 \times sin18{,}3 = 42m$
Tree is 42m

Using Pythagoras: $AB = \sqrt{((132{,}89\ldots)^2 - (41{,}7\ldots))^2}$ = 126m = width of the river

QUESTION 2

2a)

$N\hat{D}B = 360° - 208° = 152°$

$\therefore M\hat{B}D = 28°$

$B\hat{D}A = 208° - 67° = 141°$

$$\frac{\sin D\hat{B}A}{97} = \frac{\sin 141°}{120}$$

$$\therefore \sin D\hat{B}A = \frac{97 \sin 141°}{120}$$

$\therefore \sin D\hat{B}A = 0{,}5087006494$

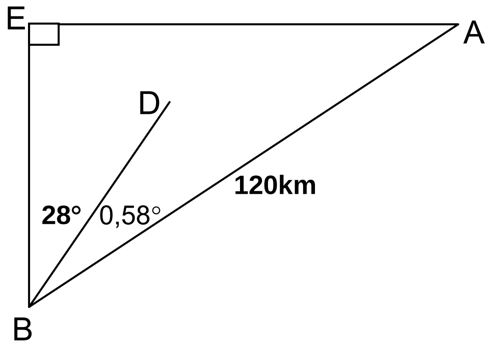
$\therefore D\hat{B}A = 30{,}58°$

$\therefore M\hat{B}A = 30{,}58° + 28°$

$\therefore M\hat{B}A = 58{,}58°$

2b)

$\hat{B} = 30{,}58°$



$$\frac{EA}{120} = \sin(28° + 30{,}58°)$$

$$EA = 120\sin(28° + 30{,}58°)$$

$$EA = 102{,}4 \quad \text{km}$$

QUESTION 3

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\therefore 7b^2 = b^2 + b^2 - 2b^2\cos A$$

$$\therefore 5b^2 = -2b^2\cos A$$

$$\therefore \frac{5b^2}{-2b^2} = \cos A$$

$$\therefore \cos A = -\frac{5}{2}$$

This equation has no solution since $-1 \le \cos A \le 1$

SESSION 24 SELF STUDY

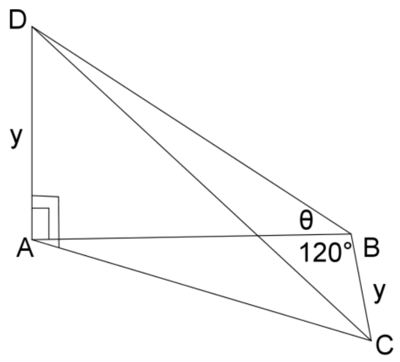
TOPIC: 3D TRIGONOMETRY

Teacher Note: Encourage learners to take time to understand all given information, including the diagram. They must fill in as much information as possible on the diagram and then work from the triangle with the most information.
Learners must revise 2D Trigonometry as they will need all the rules learned thus far to solve triangles.

SECTION A: TYPICAL EXAM TYPE QUESTIONS

QUESTION 1

In the figure A,B &C are three points in the horizontal plane, such that ∠ABC = 120˚.
D is a point directly above A
AD = BC =y
The angle of elevation of D from B is θ



a) Show that AC = $y\sqrt{\left(\frac{1}{\tan^2\theta}+\frac{1}{tan\theta}+1\right)}$ (7)

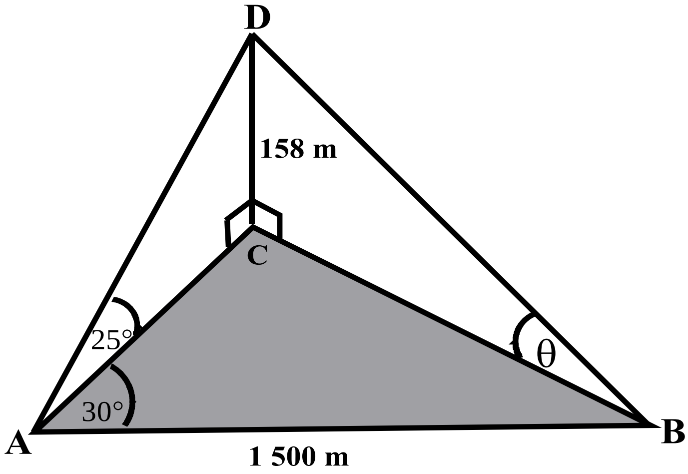
b) If y=15 and θ = 22, calculate:

i)AC (2)

ii) ∠ADC (3)
[12]

QUESTION 2

In the diagram below, AB is a straight line 1 500 m long. DC is a vertical tower 158 metres high with C, A and B points in the same horizontal plane. The angles of elevation of D from A and B are $25°$ and θ. $C\hat{A}B = 30°$.



(a) Determine the length of AC. (3)

(b) Find the value of θ. (5)

(c) Calculate the area of ΔABC. (2)

(d) Calculate the size of $A\hat{D}B$ (6)

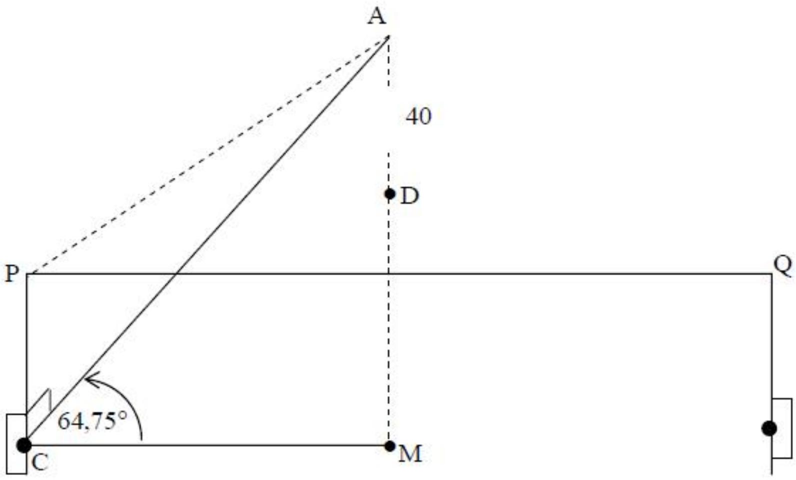
[16]

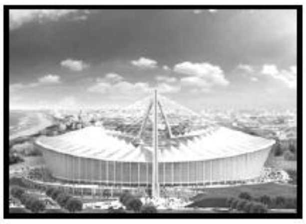
QUESTION 3

(DOE Nov 2010 P2)

The angle of elevation from a point C on the ground, at the centre of the goalpost, to the highest point A of the arc, directly above the centre of the Moses Mabhida soccer stadium, is 64,75°.

The soccer pitch is 100 metres long and 64 metres wide as prescribed by FIFA for world cup stadiums. Also AC ⊥ PC. In the figure below PQ = 100 metres and PC = 32 metres.





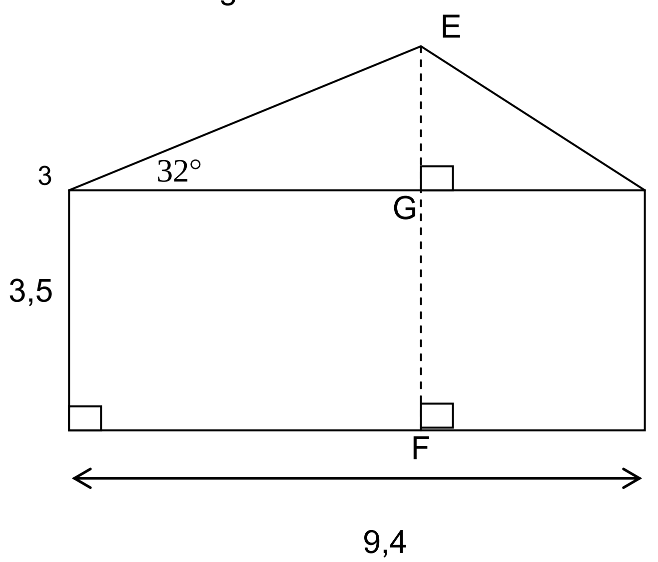
3.1 Determine AC (3)

3.2 Calculate $\angle PAC$ (3)

3.3 A camera is positioned at point D, 40 metres directly below A. Calculate the distance from D to C. (4)
[10]

QUESTION 4 *(DOE March 2011)*

The sketch below shows one side of the elevation of a house. Some dimensions (in metres are indicated on the figure.



Calculate, rounded off to ONE decimal place:

4.1 EC (3)

4.2 $D\hat{C}E$ (3)

4.3 Area of ΔDEC (2)

4.4 The height EF (3)
[11]

SECTION B: SOLUTIONS TO SECTION A

QUESTION 1

(a) In ΔADB

$\angle$ ADB= 90-θ $\tan\theta = \frac{DA}{AB} = \frac{y}{AB}$ $\therefore AB = \frac{y}{tan\theta}$

In ΔABC

$$AC^2 = AB^2 + BC^2 - 2AB.BCcos120 = \left(\frac{y}{tan\theta}\right)^2 + y^2 - 2\left(\frac{y}{tan\theta}\right)(y)(-cos60)$$

$$=y^2\frac{1}{\tan^2\theta} + y^2 + y^2\left(\frac{1}{tan\theta}\right)$$

$$= y^2[\left(\frac{1}{\tan^2\theta}\right) + 1 + \frac{1}{tan\theta}]$$

$$\therefore AC = \sqrt{y^2\left[\left(\frac{1}{\tan^2\theta}\right) + 1 + \frac{1}{tan\theta}\right]} = y\sqrt{[\left(\frac{1}{\tan^2\theta}\right) + 1 + \frac{1}{tan\theta}]}$$ (7)

(b)

(i) AC = 46,4m (2)

(ii) In ΔADC: tan(ADC) = $\frac{AC}{AD} = \frac{46,48}{15} = 3,0985$ ....

$\angle$ ADC = 72.11° (3)

[12]

QUESTION 2

(a) In Δ ADC:

$\hat{D} = 65°$ ($\angle s$ of Δ)

$$\frac{AC}{\sin 65°} = \frac{158}{\sin 25°}$$

$$\therefore AC.\sin 25° = 158.\sin 65°$$

$$\therefore AC = \frac{158.\sin 65°}{\sin 25°}$$

$$\therefore AC = 338,83\text{m}$$ (3)

(b) In Δ ACB:

$$BC^2 = 338,83^2 + 1500^2 - 2(338,83)(1500)\cos 30°$$

$$\therefore BC^2 = 1\ 484\ 499,606$$

$$\therefore BC = 1218,4\text{m}$$

In Δ DCB:

$$\tan\theta = \frac{DC}{BC}$$

$$\therefore \tan\theta = \frac{158}{1218,4}$$

$$\therefore \theta = 7,39°$$ (5)

(c) $\text{Area } \Delta ABC = \frac{1}{2}(338,83)(1500)\sin 30°$

$\therefore \text{Area } \Delta ABC = 127061,25 m^2$ (2)

(d)

$AD^2 = (338,83)^2 + (158)^2$

$\therefore AD^2 = 139769,7689$

$\therefore AD = 373,86m$

$BD^2 = (1218,4)^2 + (158)^2$

$\therefore BD^2 = 1509462,56$

$\therefore BD = 1228,60m$

$(1500)^2 = (373,86)^2 + (1228,60)^2 - 2(373,86)(1228,60)\cos A\hat{D}B$

$\therefore 2(373,86)(1228,60)\cos A\hat{D}B = (373,86)^2 + (1228,60)^2 - (1500)^2$

$\therefore 918648,792\cos A\hat{D}B = -600770,7404$

$\therefore \cos A\hat{D}B = -0,6539721661$

$\therefore A\hat{D}B = 130,84°$ (6)

[16]

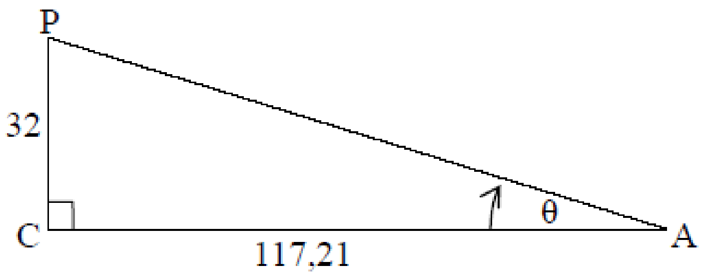
QUESTION 3

3.1 $\cos 64,75° = \frac{50}{AC}$

$AC = \frac{50}{\cos 64,75°}$

$= 117,21 \text{ m}$ (3)

3.2 PC is given to be $\frac{1}{2}(64) = 32$ m



$\tan P\hat{A}C = \frac{32}{117,21}$

$\theta = 15,27°$ (15,27042173...)

(3)

Note: If the candidate takes the unrounded answer for AC, then the answer is 15.27° (15.26987495...)

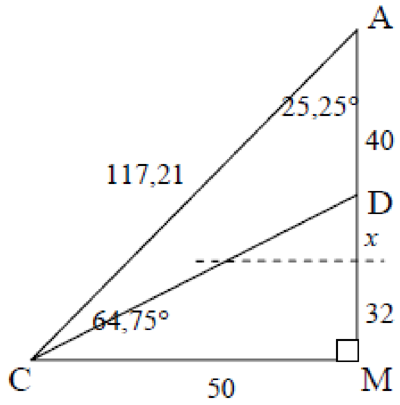
3.3 $CD^2 = 117{,}21^2 + 40^2 - 2(117{,}21)(40)\cos 25{,}25$

$= 6857{,}289092$

$\therefore CD = 82{,}81 \quad m$

Note:
If don't use the rounded off then CD = 82,81 m. Accept this answer.

OR



AM = ACsin 64,75° **OR** AM = CM tan 64,75° **OR** AM = AC cos 25,25°

$= 106{,}0111876$ $= 50 \tan 64{,}75°$ $= 117{,}21.\cos 25{,}25°$

$= 106{,}01$ $= 106{,}01$ $= 106{,}01$

$DM = 106{,}01 - 40$

$= 66{,}01$

$CD^2 = CM^2 + DM^2$

$= (50)^2 + (66{,}01)^2$

$= 6857{,}3201$ (4)

$CD = 82{,}81$ metres

[10]

QUESTION 4

4.1 $EC^2 = DE^2 + DC^2 - 2DE.DC\cos\hat{C}$

$= (7{,}5)^2 + (9{,}4)^2 - 2.(7{,}5)(9{,}4)\cos 32°$

$= 25{,}03521844...$

$EC = 5{,}0$ metres

(3)

4.2 $\dfrac{\sin DCE}{7{,}5} = \dfrac{\sin 32°}{5{,}0}$

$\sin D\hat{C}E = \dfrac{7{,}5.\sin 32°}{5{,}0}$

$= 0{,}7948788963$

$D\hat{C}E = 52{,}6°$ (3)

4.3 Area of ΔDEC

$$= \frac{1}{2}\text{DE.DC}\sin\hat{\text{D}}$$

$$= \frac{1}{2}(7{,}5)(9{,}4)\sin 32°$$

$$= 18{,}7m^2$$

OR

Area of ΔDEC

$$= \frac{1}{2}\text{CE.DC}\sin 52{,}6°$$

$$= \frac{1}{2}(5{,}0)(9{,}4)\sin 52{,}6°$$

$$= 18.7m^2$$

(2)

4.4

$$\sin 32° = \frac{\text{EG}}{7{,}5}$$

$$\text{EG} = 7{,}5.\sin 32°$$

$$= 4{,}0$$

$$\text{EF} = (4 + 3{,}5)$$

$$= 7{,}5 \text{ metres}$$

OR

$$\text{EG} = \text{EC}.\sin 52{,}6°$$

$$= (5{,}0).\sin 52{,}6°$$

$$= 4{,}0$$

$$\text{EF} = 4{,}0 + 3{,}5$$

$$= 7{,}5$$

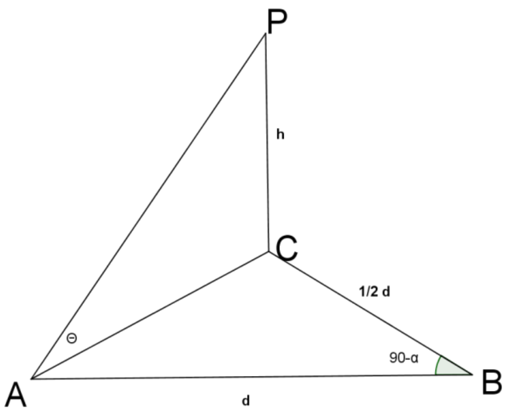
(3)

[11]

SECTION C: SOLUTIONS TO HOMEWORK

QUESTION 1

In the diagram below the points ABC lie in the same horizontal plane. A hot air balloon is stationary at point P, which is directly above C. The angle of elevation of P from A is θ. $\angle ABC$ =90 – α and the distance from B to C is half the distance of BA, where BA is d units.

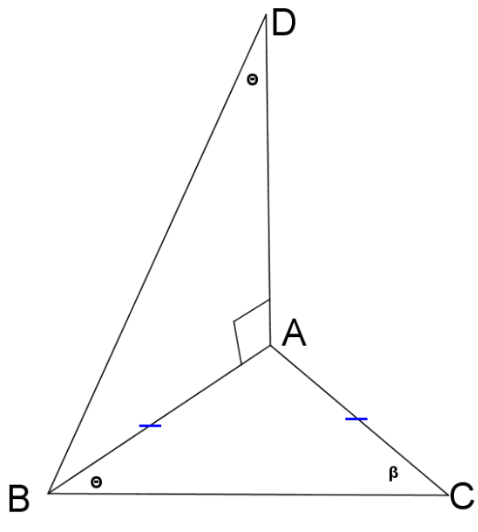


a) Show that the height of the balloon above C is $h = \frac{d\sqrt{(5-sin\alpha)}}{2}\ tan\theta$

b) Calculate h if d = 300, α =32 and θ = 63

QUESTION 2

In the figure A,B & C are three points in the same horizontal plane. AD represents a lamp pole that is perpendicular to the horizontal plane.



Given that ∠BDA = ∠ABC = θ and ∠BCA = β, BC =x

a) Write ∠BAC in terms of θ and β

b) Show that AB = $\frac{x\sin\beta}{\sin(\theta+\beta)}$

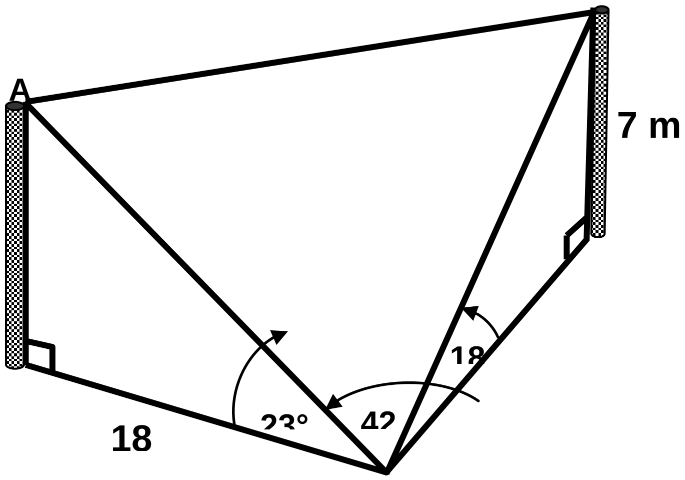
c) IF AB = AC show:

i) AB =$\frac{\quad}{2\cos\theta}$

ii) AD = $\frac{\quad}{2\sin\theta}$

QUESTION 3

Thandi is standing at point P on the horizontal ground and observes two poles, AC and BD, of different heights. P, C and D are in the same horizontal plane. From P the angles of inclination to the top of the poles A and B are 23° and 18° respectively. Thandi is 18 m from the base of pole AC. The height of pole BD is 7 m.



Calculate, correct to TWO decimal places:

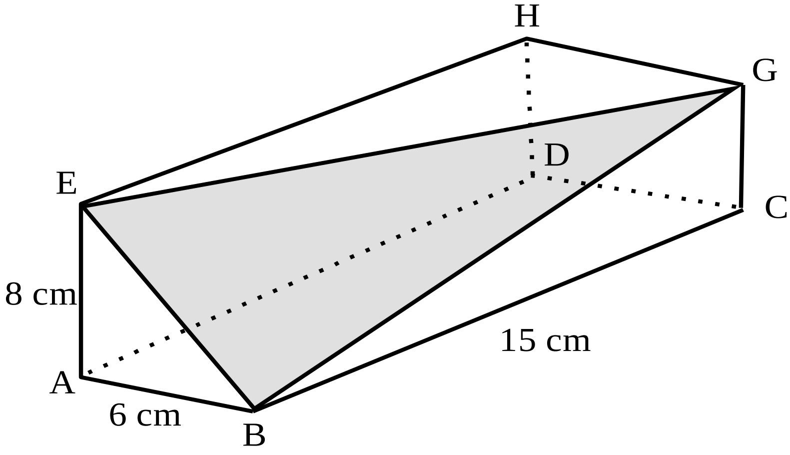
(a) The distance from Thandi to the top of pole BD.

(b) The distance from Thandi to the top of pole AC.

(c) The distance between the tops of the poles, that is the length of AB, if $A\hat{P}B = 42°$

QUESTION 4

A rectangular block of wood has a breadth of 6 metres, height of 8 metres and a length of 15 metres. A plane cut is made through the block as shown in the diagram revealing the triangular plane that has been formed. Calculate the size of $E\hat{B}G$.



SECTION D: SOLUTIONS TO HOMEWORK

QUESTION 1

a) In ΔABC

$$AC^2 = AB^2 + BC^2 - 2AB.BCcos(90-\alpha)$$

$$= d^2 + \left(\frac{1}{2}d\right)^2 - 2d(\frac{1}{2}d)sin\alpha$$

$$=\frac{5}{4}d^2 - d^2sin\alpha = d^2\left(\frac{5}{4} - sin\alpha\right)$$

$$\therefore AC = \frac{d\sqrt{(5-sin\alpha)}}{2}$$

In ΔACP

$$tan\theta = \frac{PC}{AC}$$

$$PC = h = ACtan\theta = \frac{d\sqrt{(5-sin\alpha)}}{2}tan\theta$$

b) $$h = \frac{300(\sqrt{5-4sin32})}{2}tan63 = 500m$$

QUESTION 2

a) $\angle BAC = 180 - (\theta+\beta)$

b) $\frac{AB}{sin\beta} = \frac{}{\sin(180-(\theta+\beta))}$

$AB = \frac{xsin\beta}{\sin(\theta+\beta)}$

c i) IF AB = AC Then θ = β

$AB = \frac{xsin\theta}{sin2\theta} = \frac{xsin\theta}{2sin\theta cos\theta} = \frac{}{2cos\theta}$

ii) In ΔBDA

$\angle B = 90 - \theta$

$\frac{AB}{sin\theta} = \frac{AD}{\sin(90-\theta)}$ $\therefore AD = \frac{cos\theta(\frac{x}{2cos\theta})}{sin\theta} = \frac{}{2sin\theta}$

QUESTION 3

a)

$$\frac{7}{PB} = \sin 18°$$

$$\therefore PB = \frac{7}{\sin 18°}$$

$$\therefore PB = 22,65247584..$$

b)

$$\frac{18}{PA} = \cos 23°$$

$$\therefore PA = \frac{18}{\cos 23°}$$

$$\therefore PA = 19,55448679....$$

c)

$$AB^2 = (22,65)^2 + (19,55)^2 - 2(22,65)(19,55).\cos 42°$$

$$\therefore AB^2 = 237,0847954...$$

$$\therefore AB = 15,40 \text{ m}$$

QUESTION 4

In Δ AEB:

$EB^2 = 8^2 + 6^2$

$\therefore EB^2 = 100$

$\therefore EB = 10$

In Δ GBC:

$BC^2 = 15^2 + 8^2$

$\therefore BC^2 = 289$

$\therefore BC = 17$

In Δ ACB:

$EG^2 = 15^2 + 6^2$

$\therefore EG^2 = 261$

$\therefore EG = \sqrt{261}$

In Δ EGB:

$\therefore \left(\sqrt{261}\right)^2 = 17^2 + 10^2 - \left(2(17)(10)\cos E\hat{B}G\right)$

$\therefore 261 = 389 - \left(340\cos E\hat{B}G\right)$

$\therefore -128 = -340\cos E\hat{B}G$

$\therefore \dfrac{32}{85} = \cos E\hat{B}G$

$\therefore E\hat{B}G = 67,88°$