SENIOR SECONDARY IMPROVEMENT PROGRAMME 2013

education
Department: Education
GAUTENG PROVINCE

GRADE 12

MATHEMATICS

LEARNER NOTES

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LEARNER NOTES

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SESSION 16

TOPIC: DATA HANDLING

Learner Note: Data Handling makes up approximately 20% of paper two. This session is designed in particular to help you understand how to apply what you have learnt in grade 11 to answer questions regarding best fit and distribution of data. It is important that you understand that it is crucial that you are able to interpret a set of data and communicate that.

SECTION A: TYPICAL EXAM QUESTIONS

QUESTION 1

The ages of the final 23 players selected by coach Carlos Perreira to play for Bafana Bafana in the 2010 FIFA World Cup are provided on the following page.

Position	Player	Age
1	Shu-Aib Walters	28
2	Siboniso Gaxa	26
3	Tshepo Masilela	25
4	Aaron Moekoena	29
5	Lucas Thwala	28
6	Macbeth Sibaya	32
7	Lance Davids	25
8	Siphiwe Tshabalala	25
9	Katlego Mphela	25
10	Steven Pienaar	28
11	Teko Modise	27
12	Reneilwe Letsholonyane	28
13	Kagisho Dikgacoi	25
14	Matthew Booth	33
15	Bernard Parker	24
16	Itumeleng Khune	22
17	Surprise Moriri	30
18	Siyabonga Nomvethe	32
19	Anele Ngcongca	22
20	Bongani Khumalo	23
21	Siyabonga Sangweni	28
22	Moeneeb Josephs	30
23	Thanduyise Khuboni	24

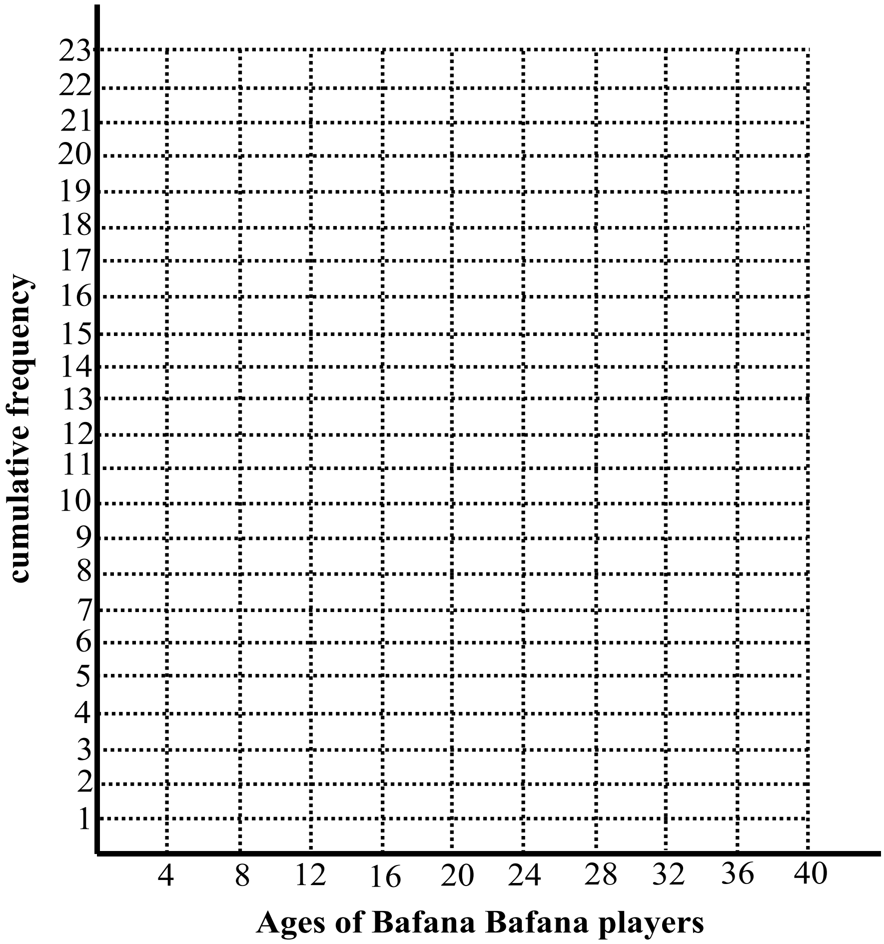
Source: www.2010 Fifa World Cup:final squads – MediaClubSouthAfica.com

The ages of the players are to be grouped into class intervals.

(a) Complete the following table: (2)

Class intervals (ages)	Frequency	Cumulative frequency
$16 \leq x < 20$		
$20 \leq x < 24$		
$24 \leq x < 28$		
$28 \leq x < 32$		
$32 \leq x < 36$		

(b) On the diagram provided below, draw a cumulative frequency curve for this data. (6)



(c) Use your graph to read off approximate values for the quartiles. (3)

[11]

QUESTION 2

(a) Complete the table and then use the table to calculate the standard deviation. (5)

Class intervals	Frequency (f)	Midpoint (m)	$f \times m$	$m - \overline{x}$	$(m-\overline{x})^2$	$f \times (m-\overline{x})^2$
$20 \leq x < 24$	3	22				
$24 \leq x < 28$	9	26				
$28 \leq x < 32$	8	30				
$32 \leq x < 36$	3	34				
			$\overline{x} =$			

(b) Hence calculate the standard deviation using the table. (2)

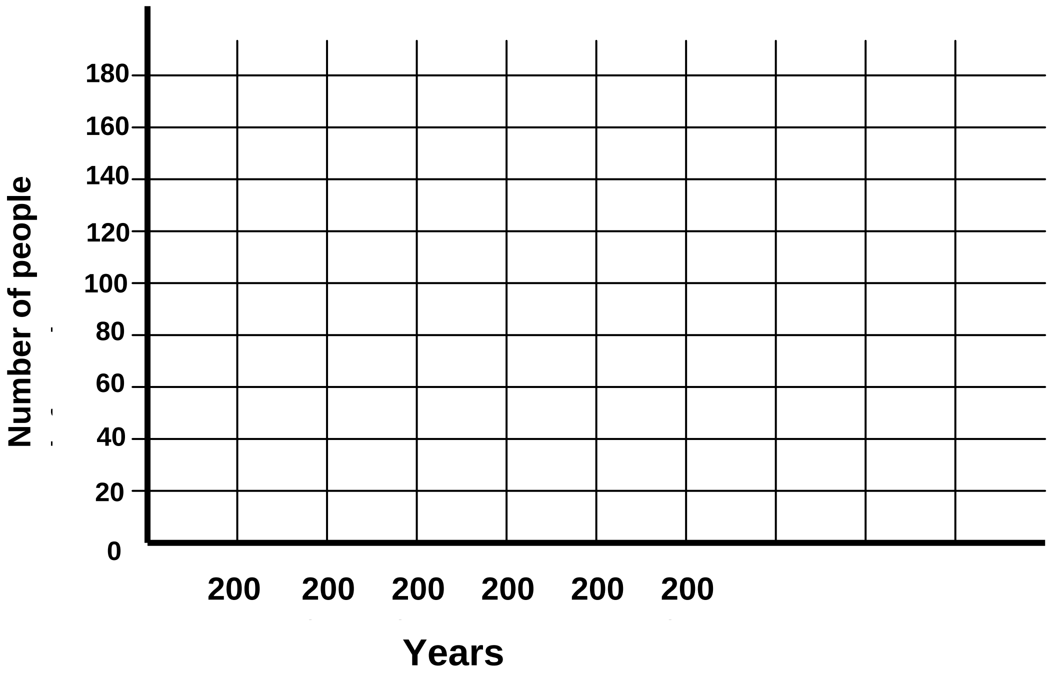
(c) Now use your calculator to verify your answer. (2)
[9]

QUESTION 3

The table below represents the number of people infected with malaria in a certain area from 2001 to 2006:

YEAR	NUMBER OF PEOPLE INFECTED
2001	117
2002	122
2003	130
2004	133
2005	135
2006	137

(a) Draw a scatter plot to represent the above data. Use the diagram provided below.
(2)



(b) Explain whether a linear, quadratic or exponential curve would be a line of best fit for the above-mentioned data. (1)

(c) If the same trend continued, estimate, by using your graph, the number of people who will be infected with malaria in 2008. (1)
[4]

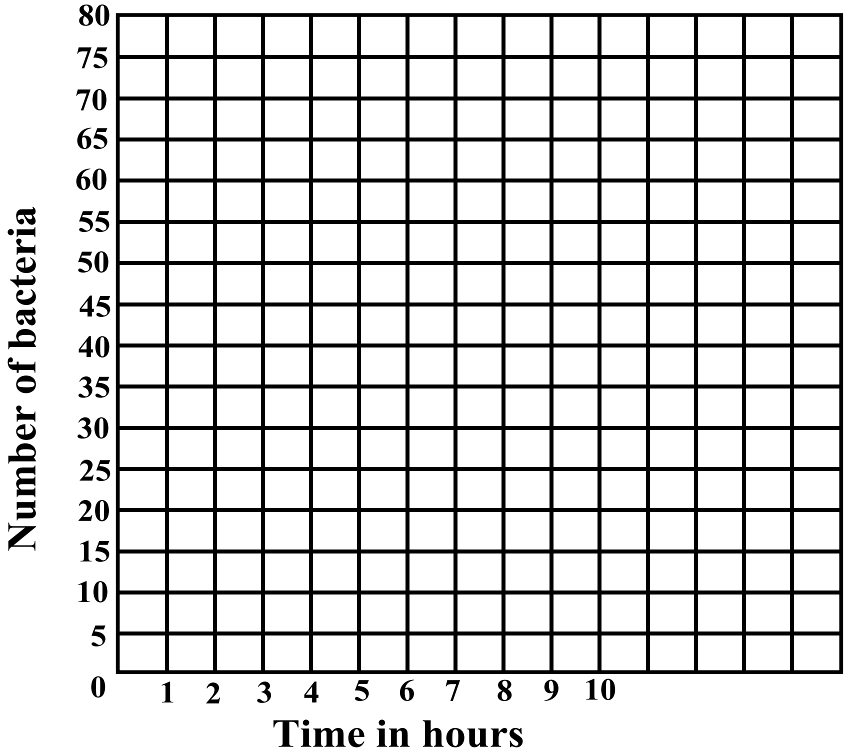
QUESTION 4

A medical researcher recorded the growth in the number of bacteria over a period of 10 hours. The results are recorded in the following table:

Time in hours	0	1	2	3	4	5	6	7	8	9	10
Number of bacteria	5	10	7	13	10	20	30	35	45	65	80

(a) On the diagram provided below, draw a scatter plot to represent this data. (2)

(b) State the type of relationship (linear, quadratic or exponential) that exists between the number of hours and the growth in the number of bacteria. (1)



(3)
[6]

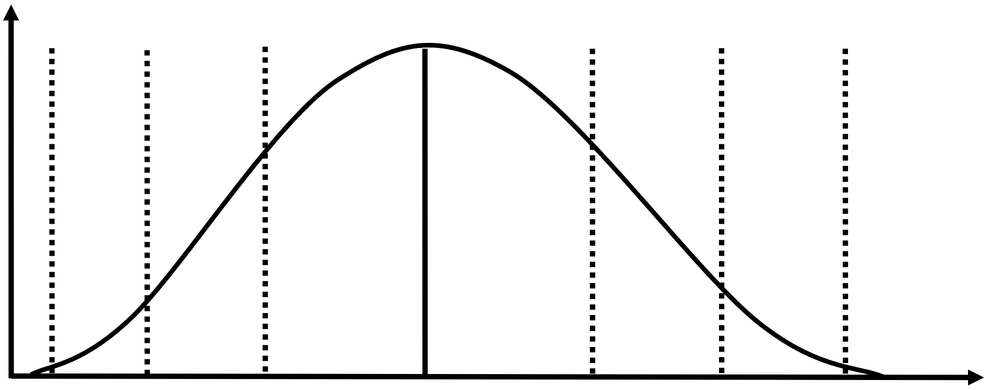
QUESTION 5

The duration of telephone calls made by a receptionist was monitored for a week. The data obtained is represented by the normal distribution curve on the following page. The mean time was 176 seconds with a standard deviation of 30 seconds.

(a) What percentage of calls made was between 146 seconds and 206 seconds in duration? Fill in the necessary information on the graph provided below. (2)

(b) Determine the time interval for the duration of calls for the middle 95% of the data. (2)

(c) What percentage of calls made were in excess of 146 seconds? (2)



[6]

SECTION B – ADDITIONAL CONTENT NOTES

Mean

The mean of a set of data is the average. To get the mean, you add the scores and divide by the number of scores.

Mode

This is the most frequently occurring score.

Quartiles

Quartiles are measures of dispersion around the median, which is a good measure of central tendency. The median divides the data into two halves. The lower and upper quartiles further subdivide the data into quarters.

There are three quartiles:

The Lower Quartile (Q_1): This is the median of the lower half of the values.

The Median (M or Q_2): This is the value that divides the data into halves.

The Upper Quartile(Q_3): This is the median of the upper half of the values.

If there is an odd number of data values in the data set, then the specific quartile will be a value in the data set. If there is an even number of data values in the data set then the specific quartile will not be a value in the data set. A number which will serve as a quartile will need to be inserted into the data set (the average of the two middle numbers).

Range

The range is the difference between the largest and the smallest value in the data set. The bigger the range, the more spread out the data is.

The Inter-quartile range (IQR)

The difference between the lower and upper quartile is called the inter-quartile range.

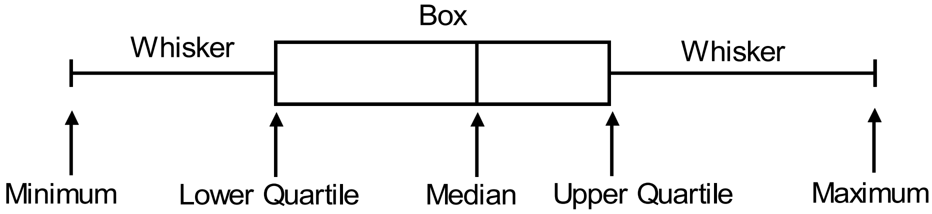
Five Number Summaries

The Five Number Summary uses the following measures of dispersion:

- Minimum: The smallest value in the data
- Lower Quartile: The median of the lower half of the values
- Median: The value that divided the data into halves
- Upper Quartile: The median of the upper half of the values
- Maximum: The largest value in the data

Box and Whisker Plots

A Box and Whisker Plot is a graphical representation of the Five Number Summary.



Standard deviation and variance

Standard deviation and variance are a way of measuring the spread of a set of data. These values also tell us how each value digresses from the mean value. It is important that learners understand what these two concepts are so that they are able to interpret their results and communicate conclusions. Learners need to know how to calculate standard deviation manually using a table as well as their calculators.

The standard deviation (SD) can be determined by using the following formula:

$$\text{SD} = \sqrt{\frac{\sum (x-\bar{x})^2}{n}} \quad ariance = \frac{\Sigma(x-\bar{x})^2}{n}$$

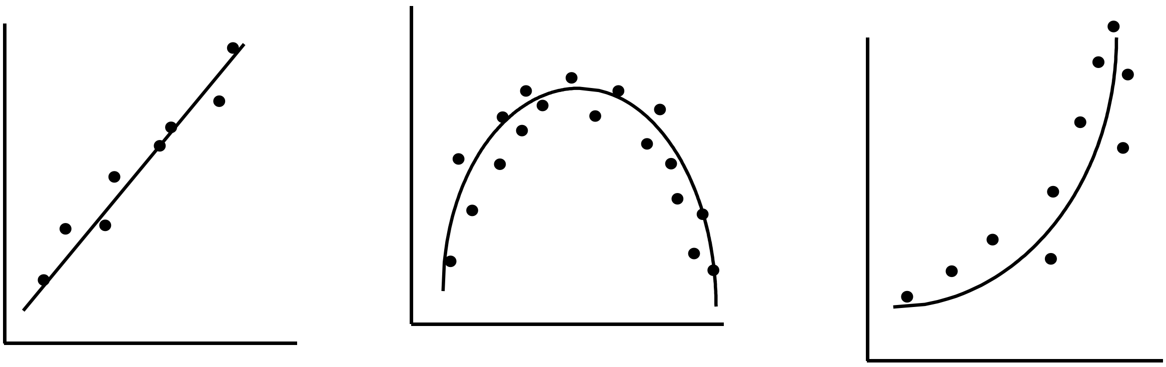
Calculator programmes to calculate standard deviation

CASIO fx-82ES PLUS:
MODE
2 : STAT
1 : 1 – VAR
Enter the data points: push = after each data point
AC
SHIFT
STAT
5: VAR
3 : $x\,\sigma\,n$
push = to get standard deviation

SHARP DAL
MODE 1 =
Enter data points: push STO 2 M+ after each data point
RCL 6 to get standard deviation

Scatter plots and lines and curves of best fit

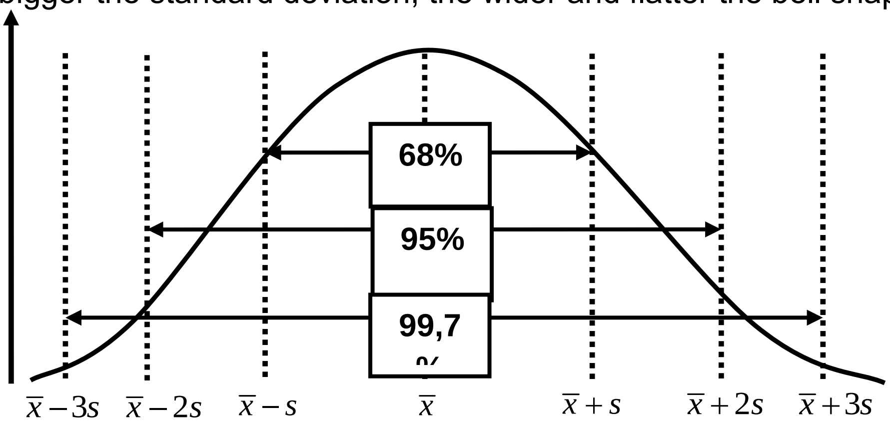
Plotting data on a scatter plot diagram will show trends in the data. Data could follow a linear, quadratic or exponential trend.



Linear Quadratic Exponential

The normal distribution curve

1. The mean, median and mode have the same value.
2. An equal number of scores lie on either side of the mean.
3. The majority of scores (99,7%) lie within three standard deviations from the mean, i.e. in the interval $(\overline{x}-3s\,;\overline{x}+3s)$ where $\overline{x}$ represents the mean and s represents the standard deviation.
4. About 95% of scores lie within two standard deviations from the mean, i.e. in the interval $(\overline{x}-2s\,;\overline{x}+2s)$ where $\overline{x}$ represents the mean and s represents the standard deviation.
5. About two-thirds of scores (68%) lie within one standard deviation from the mean, i.e. in the interval $(\overline{x}-s\,;\overline{x}+s)$ where $\overline{x}$ represents the mean and s represents the standard deviation.
6. The smaller the standard deviation, the thinner and taller the bell shape is. The bigger the standard deviation, the wider and flatter the bell shape is.



SECTION C: HOMEWORK

QUESTION 1

The ages of the final 23 players selected by coach Oscar Tabarez to play for Uruguay in the 2010 FIFA World Cup are provided below.

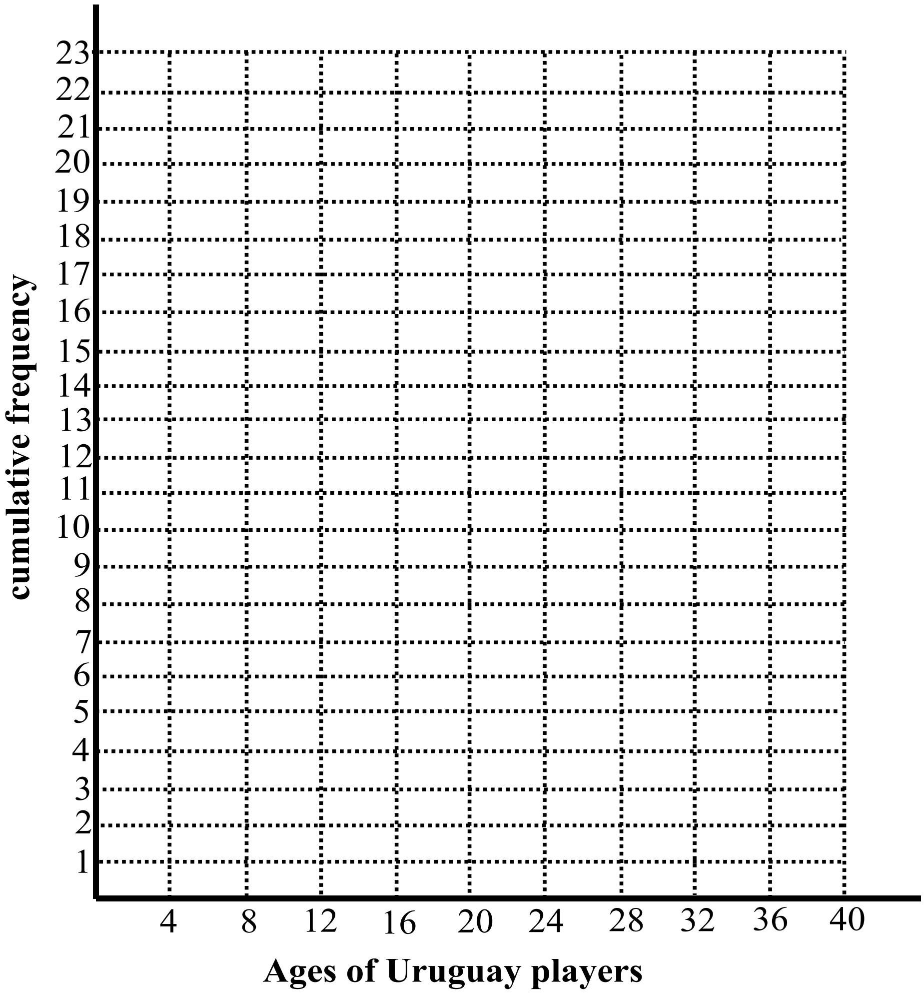
Position	Player	Age
1	Fernando Musiera	23
2	Diego Lugano (captain)	29
3	Diego Godin	24
4	Jorge Fucile	25
5	Walter Gargano	25
6	Andres Scotti	35
7	Edinson Cavani	23
8	Sebastian Eguren	29
9	Luis Suarez	23
10	Diego Forlan	31
11	Alvaro Perreira	25
12	Juan Castillo	32
13	Sebastian Abreu	33
14	Nicolas Lodeira	23
15	Diago Perez	30
16	Maxi Perrreira	26
17	Ignacio Gonzales	28
18	Egidio Arevalo Rios	27
19	Sebastian Fernandes	25
20	Mauricio Victorino	27
21	Alvaro Fernandez	24
22	Martin Caceres	23
23	Martin Silva	27

Source: www.2010 Fifa World Cup:final squads – MediaClubSouthAfica.com

(a) Complete the following table: (2)

Class intervals (ages)	Frequency	Cumulative frequency
$16 \leq x < 20$		
$20 \leq x < 24$		
$24 \leq x < 28$		
$28 \leq x < 32$		
$32 \leq x < 36$		

(b) On the diagram provided below, draw an ogive representing the above data. (6)



(c) Use your graph to read off approximate values for the quartiles. (3)
[11]

QUESTION 2

(a) Complete the table and then use the table to calculate the standard deviation. (5)

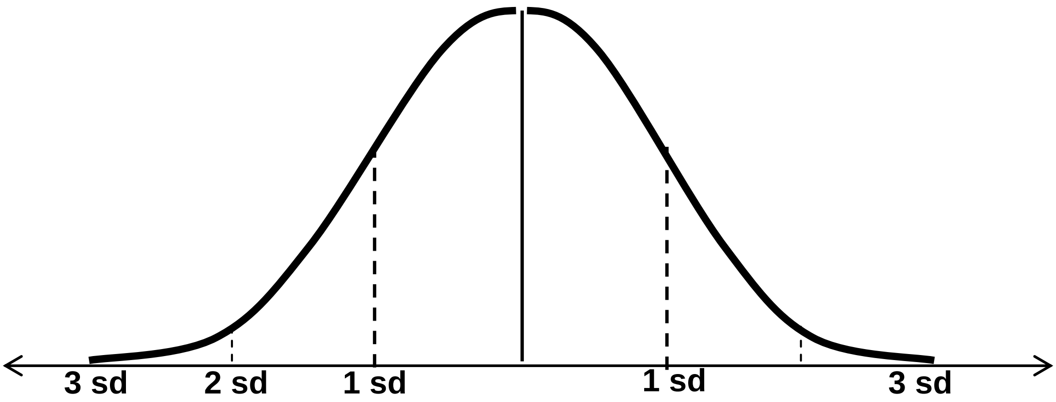
Class intervals	Frequency (f)	Midpoint (m)	$f \times m$	$m - \overline{x}$	$(m-\overline{x})^2$	$f \times (m-\overline{x})^2$
$20 \leq x < 24$	3	22				
$24 \leq x < 28$	9	26				
$28 \leq x < 32$	8	30				
$32 \leq x < 36$	3	34				
			$\overline{x} =$			

(b) Hence calculate the standard deviation using the table. (2)

(c) Now use your calculator to verify your answer. (2)
[9]

QUESTION 3

After protracted union protests, a company analysed its salary structure for employees. They found that the salaries were symmetrically distributed with a mean of R8 850 per month and a standard deviation of R2 950 per month. Research indicated that if the monthly salary was below R3 000, the employee would not maintain an acceptable quality of life.



(a) Estimate the percentage of employees who will struggle to maintain an acceptable quality of life. (2)

(b) Estimate the percentage of employees who earn more than R11 800 per month. (1)

(c) Do you think that the company has a fair salary structure? Use the given data to motivate your answer. (1)

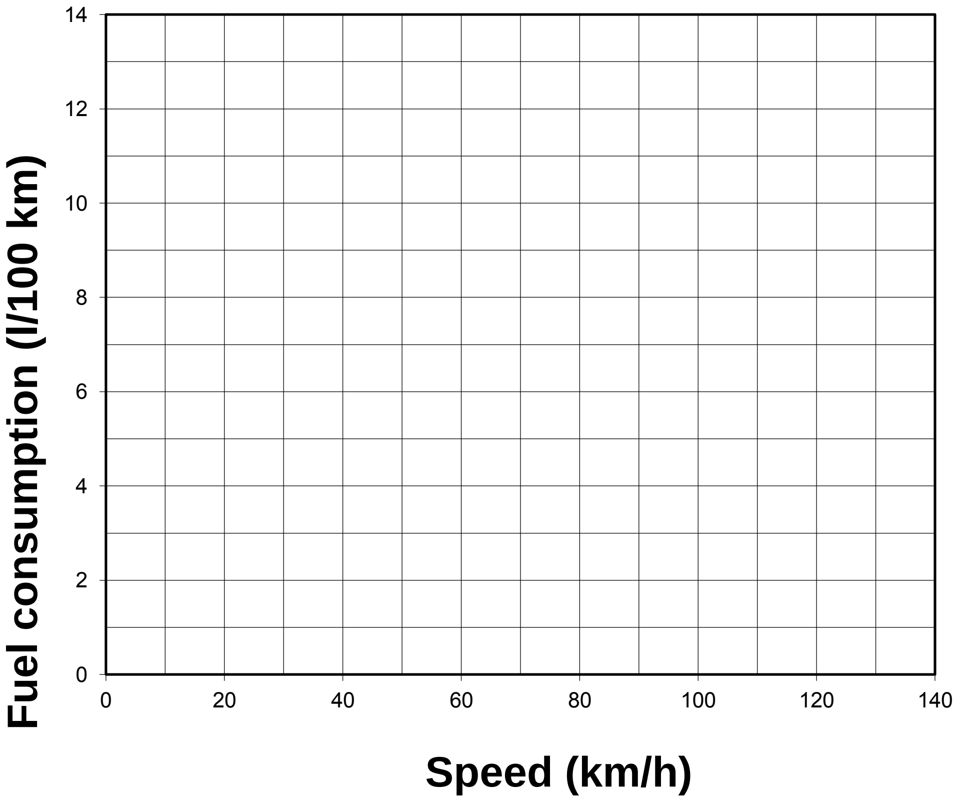
[4]

QUESTION 4

A motor company did research on how the speed of a car affects the fuel consumption of the vehicle. The following data was obtained:

Speed in km/h	60	75	115	85	110	95	120	100	70
Fuel consumption in ℓ/100 km	11,5	10	8,4	9,2	7,8	8,9	8,8	8,6	10,2

(a) Represent the data as a scatter plot on the diagram provided. (3)



(b) Suggest whether a linear, quadratic or exponential function would best fit the data. (1)

(c) What advice can the company give about the driving speed in order to keep the cost of fuel to a minimum? (2)

[6]

SECTION D: SOLUTIONS AND HINTS TO SECTION A

QUESTION 1

(a)

Class intervals (ages)	Frequency ✓	Cumulative frequency ✓
$16 \le x < 20$	0	0
$20 \le x < 24$	3	3
$24 \le x < 28$	9	12
$28 \le x < 32$	8	20
$32 \le x < 36$	3	23

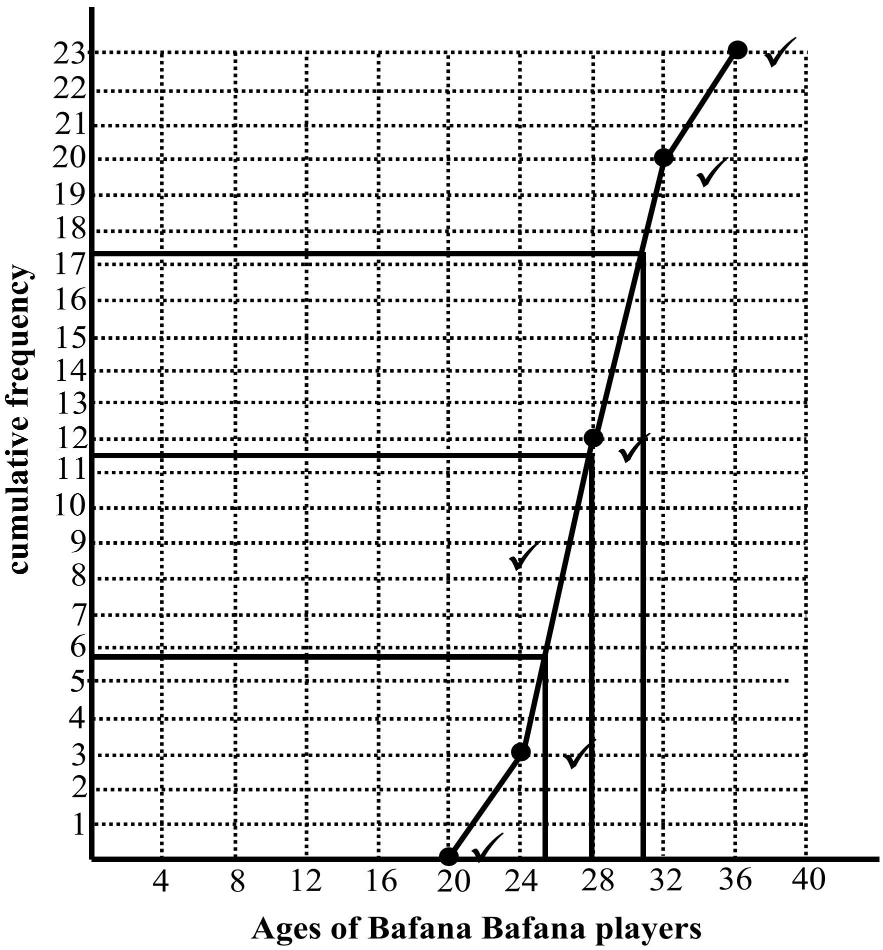
(2)

(b)

Class intervals (ages)	Frequency	Cumulative frequency	Graph points
$16 \leq x < 20$	0	0	$(20;0)$
$20 \leq x < 24$	3	3	$(24;3)$
$24 \leq x < 28$	9	12	$(28;12)$
$28 \leq x < 32$	8	20	$(32;20)$
$32 \leq x < 36$	3	23	$(36;23)$

Learner Note: Cumulative frequency graphs can also be referred to as ogives!

Learner Note: Write down the co-ordinates. X coordinate is the last number in the interval and the y value is the cumulative frequency



(6)

(c)

Lower quartile $23 \times \frac{1}{4} = 5.75$ (5,75;25) Therefore Q_1 = 25✓ **Median** $23 \times \frac{1}{2} = 11.5$ (11.5;28) Therefore Median = 28✓ **Upper quartile** $23 \times \frac{3}{4} = 17.25$ (17.25;31) Therefore Q_3 = 31 ✓	**Learner Note:** Lower quartile is the 25th percentile thus multiply the cumulative frequency by a quarter and read off the graph to determine the y-value. Similarly the median is the 50th percentile thus multiply the cumulative frequency by a half and the upper quartile is determined by multiplying the cumulative frequency by three quarters (the 75th percentile) (3)

[11]

QUESTION 2

(a)

Class intervals	Frequency (f)	Midpoint (m)	$f \times m$ ✓	$m-\bar{x}$ ✓	$(m-\bar{x})^2$ ✓	$f \times (m-\bar{x})^2$ ✓
$20 \le x < 24$	3	22	66	–5,9	34,81	104,43
$24 \le x < 28$	9	26	234	–1,9	3,61	32,49
$28 \le x < 32$	8	30	240	2,1	4,41	35,28
$32 \le x < 36$	3	34	102	6,1	37,21	111,63
			$\bar{x} = \frac{642}{23} = 27,9$ ✓			$\sum f \times (m-\bar{x})^2 = 283,83$

(5)

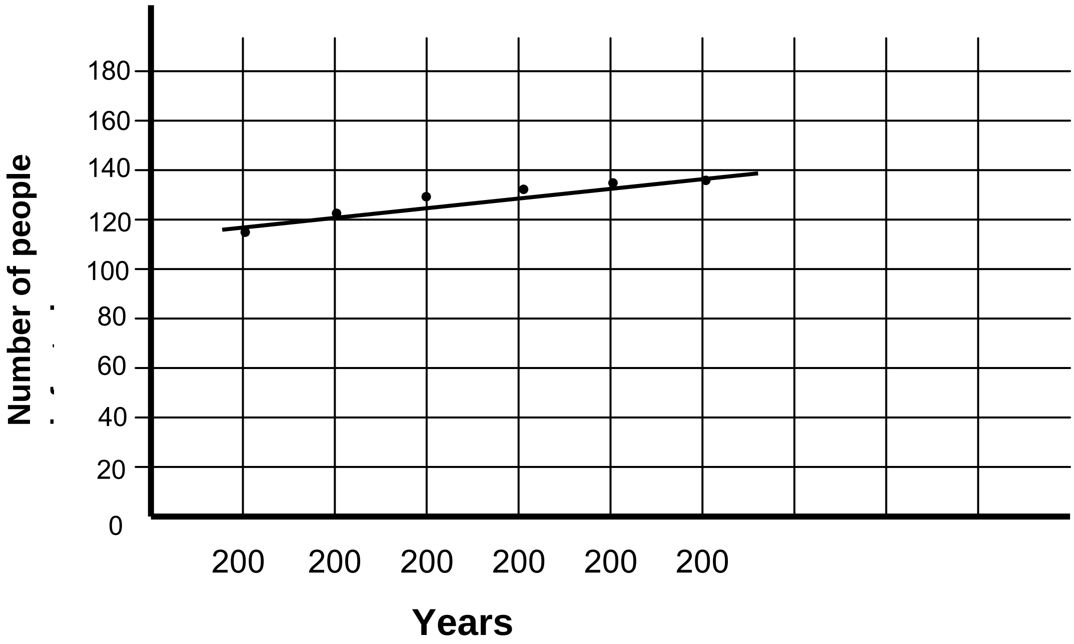
Learner Note: Remember mean can be calculated by multiplying the midpoint of the interval and relative frequency.

(b) $\text{SD} = \sqrt{\frac{\sum f.(x-\bar{x})^2}{23}} = \sqrt{\frac{283,83}{23}} = 3,5$	✓✓ (2)

CASIO fx-82ES PLUS: MODE 2 : STAT 1 : 1 – VAR SHIFT SETUP 3: STAT (you need to scroll down to get this function) 1: ON Enter the midpoints: 22= 26= 30= 34= Enter the frequencies: 3= 9= 8= 3= AC SHIFT 1 4: VAR 3 : $x\sigma n$ = The answer will read:3,5 **SHARP DAL:** MODE 1= Enter data: 22 STO 3 M+ 26 STO 9 M+ 30 STO 8 M+ 34 STO 3 M+ RCL 6 to get 3,5	✓✓ (2) **[9]**

QUESTION 3

(a)

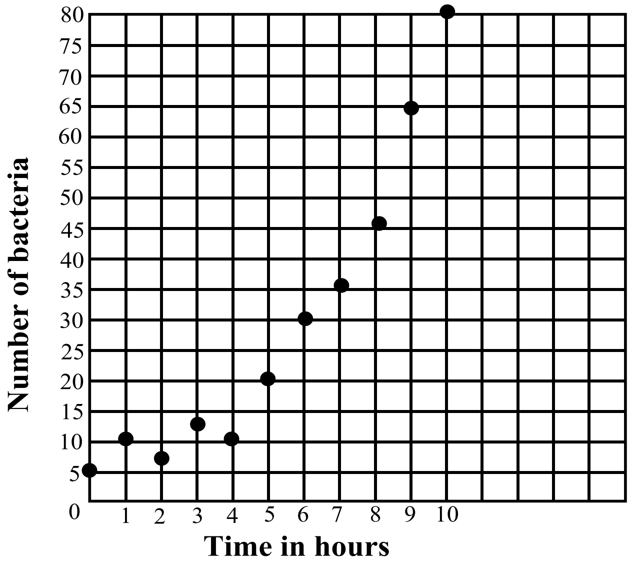


(2)

b. Linear	✓
c. Approx. 145 people	✓

[4]

QUESTION 4



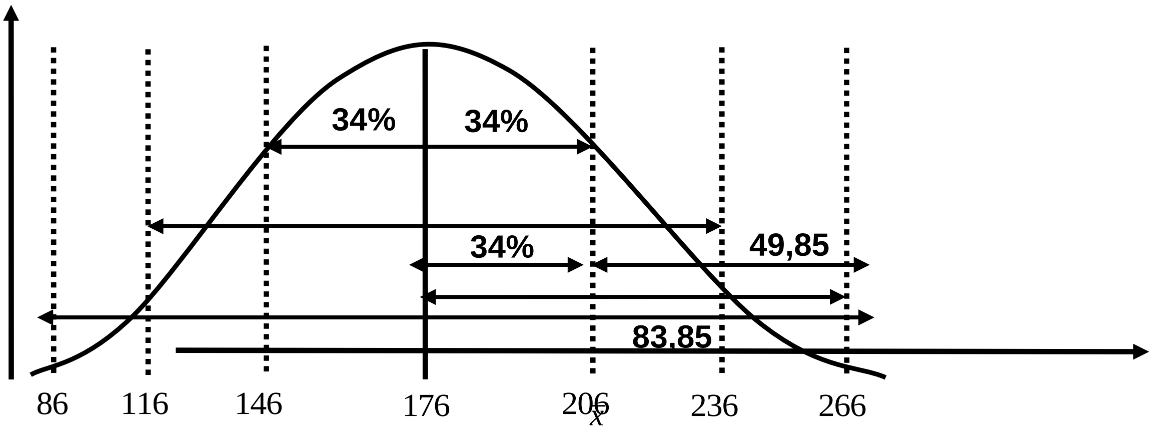
✓✓

(2)

b. Quadratic or exponential	✓ **Learner Note:** If ever asked to describe or state the relationship, answer by identifying the function, e.g. linear, quardratic, exponential. (1)

[3]

QUESTION 5



One standard deviation interval:

$(\bar{x}-s\,;\,\bar{x}+s)$

$=(176-30;176+30)$

$=(146;206)$

Two standard deviation intervals:

$(\bar{x}-s\,;\,\bar{x}+s)$

$=(176-2(30);176+2(30))$

$=(116;236)$

Three standard deviation intervals:

$(\bar{x}-s\,;\,\bar{x}+s)$

$=(176-3(30);176+3(30))$

$=(86;266)$

a. The interval between 146 seconds and 206 seconds lies between one standard deviation of the mean. For the normal distribution, approximately 68% of the data lies between one standard deviation of the mean.	✓✓
b. The middle 95% of the data for a normal distribution lies between two standard deviations on either side of the mean. The middle 95% of the calls will be between 116 and 236 seconds.	✓✓
c. Approximately 34% of the calls are between 146 and 176 seconds. Another 49, 85% of the calls are in excess of 176 seconds. Therefore, in total, approximately 84% of the calls are in excess of 146 seconds.	✓✓ [6]

SESSION 16.2

TOPIC: TRANSFORMATIONS

Learner Note: You must revise all previous grades work on Transformations. The focus in grade 12 will be on rotations through any angle θ however you will be examined on all the work you've learned thus far on Transformations.

SECTION A: TYPICAL EXAM QUESTIONS

QUESTION 1

Determine the coordinates of the image of point A(-2; $-\sqrt{3}$) after a clockwise rotation, about the origin through an angle of 210°. [6]

Hint: Be careful with your signs, as this is a clockwise rotation.

QUESTION 2

Consider a square with coordinates P(0; -5), Q(5;0), R(0;5) and S(-5;0) and answer the following questions: (12)

2.1 If P'Q'R'S' is the reflection of PQRS about the line y = -x give the coordinates of P' and R'. (4)

2.2 Calculate, correct to two decimal places, the coordinates of P'' and R'' if P'Q'R'S' is rotated about the origin through an angle of 45° anticlockwise to give P''Q''R''S''. (4)

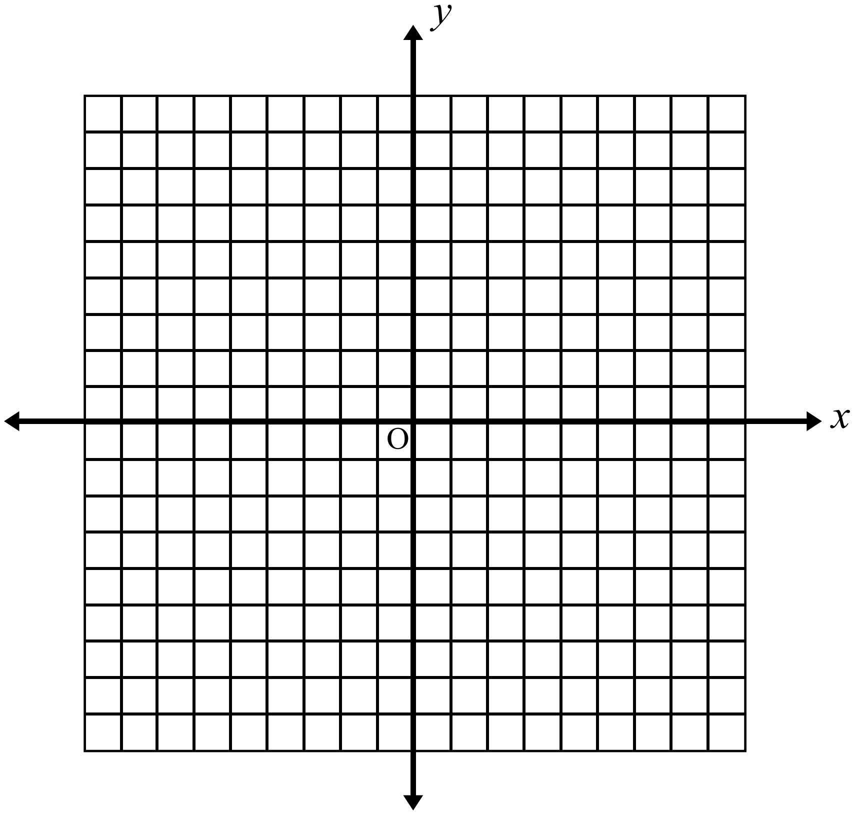
2.3 If S is rotated anticlockwise about the origin through 60° to a new position S''', find the coordinates of S''' without a calculator – leave your answer on surd form. (4)

[24]

QUESTION 3

3.1 On the diagram provided below, draw figure ABCD with the coordinates of the vertices as follows: A (-6;6), B (-4;2), C (-2;6) and D(-4;8). (1)

3.2 On the same diagram, draw the image A'B'C'D' if ABCD is rotated 90°anti-clockwise. Indicate the coordinates of B' and C' (2)



3.3 State the general rule in terms of *x* and *y* of the rotation 90°anti-clockwise, using the notation (x; y) → (1)

3.4 Now draw the image A'B'C'D' if ABCD is transformed under the rule:
$(x; y) \rightarrow (\frac{1}{2}x; \frac{1}{2}y)$. (2)

3.5 Write down the value of the following ratio: $\frac{\text{Area ABCD}}{\text{Area A'B'C'D'}}$ (2)

3.6 Draw image A"B"C"D" if A'B'C'D' is rotated 180° clockwise. Indicate the coordinates of A"'. (2)

3.7 Draw the image EFGH if A'B'C'D' is transformed under the rule (x, y) → (-x; y). Indicate the coordinates of E. (2)

3.8 Write down the single algebraic rule if ABCD is reflected about the *x*-axis, followed by a translation of 7 units right, followed by an enlargement by a scale factor of 2 units. (4)
[16]

QUESTION 4

A circle with equation $x^2 + {}^2 - 2x - 4y = 4$ is rotated 90° anticlockwise about the origin and then enlarged by a scale factor of 2, find the new equation. [6]

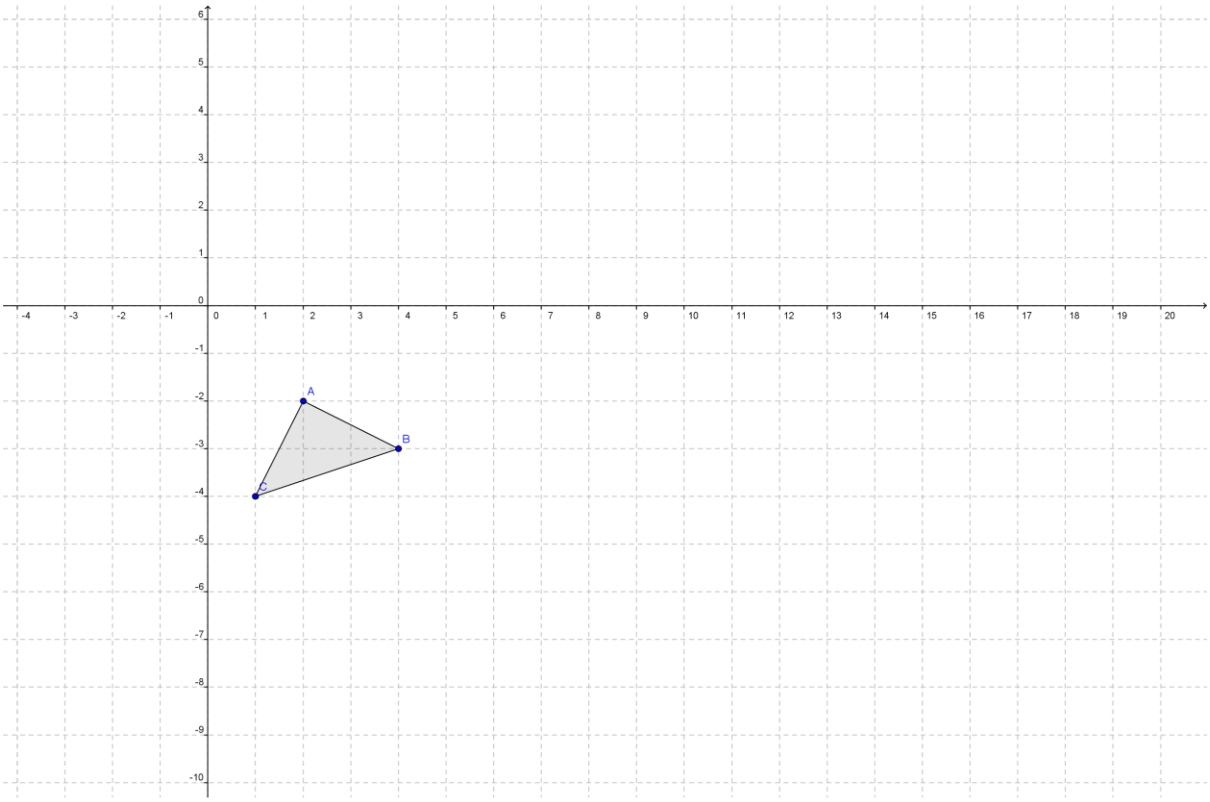
Hint: Rule for rotation through 90° anticlockwise about the origin (x, y) → (-y; x).

QUESTION 5 *(DOE Nov 2010 P2)*

A transformation T is described as follows:

- A reflection in the *x*-axis, followed by
- A translation of 4 units left and 2 units down, followed by
- An enlargement through the origin by a factor of 2

In the diagram Δ ABC is given with vertices A(2 ; –2), B(4 ; –3) and C(1 ; –4).



5.1 If ΔABC is transformed by T to A'B'C' (in that order), on the same system of axes sketch ΔA'B'C'. Show ALL the steps. (6)

5.2 Write down the general rule for (x; y) under transformation T in the form (*x* ; *y*) → (4)

5.3 Calculate the area of ΔA'B'C'. (4)

[14]

QUESTION 6 *(DOE March 2011)*

Consider the point A (– 12 ; 6). The point is reflected about the x-axis to A'

6.1 Write down the coordinates of A'. (1)

6.2 An alternative transformation from A to A' is a rotation about the origin through α°, where α ∈ [0; 90] Calculate α. (6)

[7]

SECTION B: HOMEWORK

QUESTION 1

Point P(2; 4) is rotated about the origin through an obtuse angle θ, in an anticlockwise

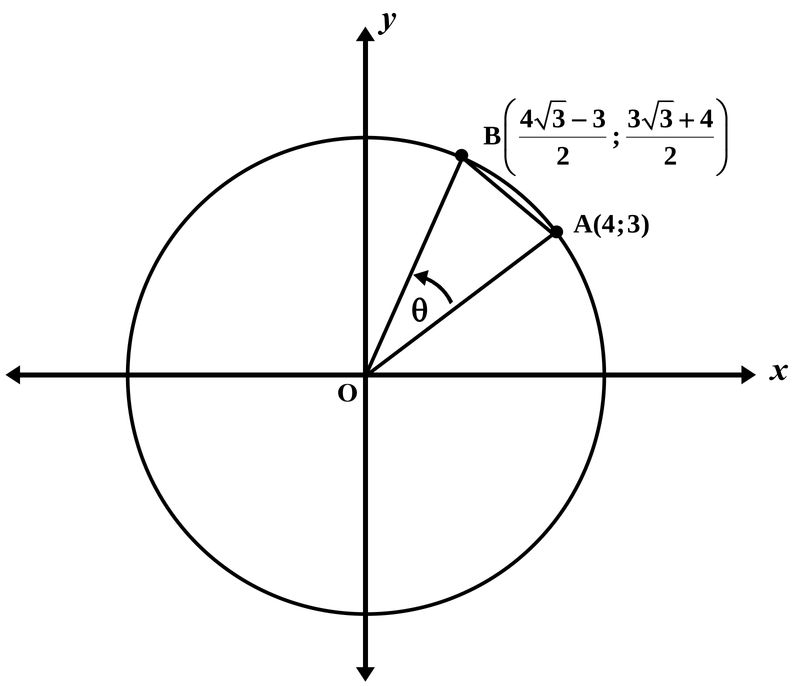
direction. The image is X(-3 2; y), y<0.

Find:

a) The value of y √ b) The angle θ

QUESTION 2

In the diagram below, the circle with centre the origin is rotated anti-clockwise about the origin through an angle of θ degrees. Point $A(4\,;3)$ lies on the circle and the image of point A is point B (coordinates are indicated on the diagram).



2.1 Determine the length of the radius of the circle

2.2 Calculate the size of angle θ

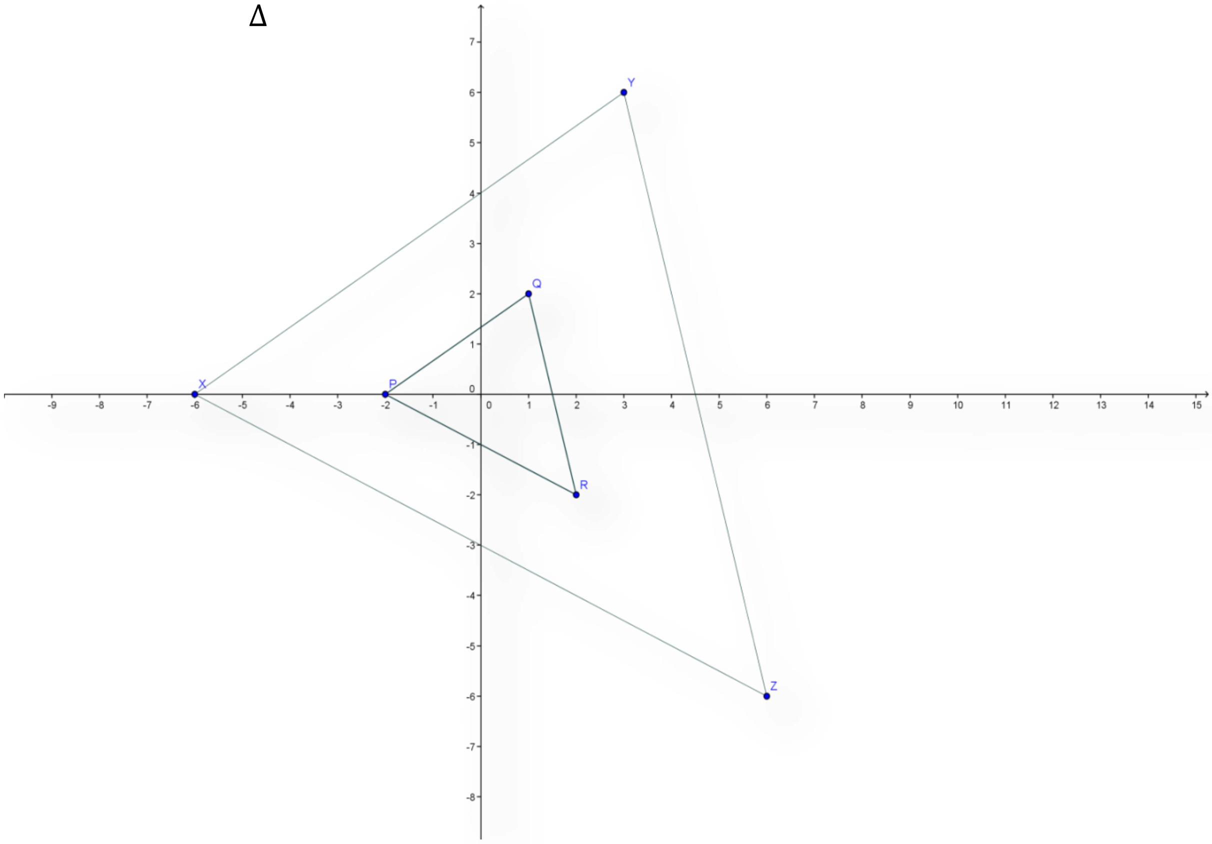
2.3 Hence show that $AB = 5\sqrt{2-\sqrt{3}}$

2.4 Calculate the area of ΔOAB

QUESTION 3

In the diagram below PQR has been enlarged through the origin by a scale factor of 3 to give its image XYZ. Δ

Δ



3.1 Write down the coordinates of XYZ.

3.2 Calculate $\hat{Y}$ (round off your answer to one decimal place)

SECTION C: SOLUTIONS TO SECTION A

QUESTION 1

x' = $x_A cos\theta$ - $_A sin\theta$ = -2cos(-210) – (-$\sqrt{3}$)sin(-210)
= -2cos(-210+360) + $\sqrt{3}$sin(-210+360) (Add 360 to make angles positive)
= -2cos150 + $\sqrt{3}$sin150
=-2cos(180-30) + $\sqrt{3}$sin(180-30)
= 2cos30 +$\sqrt{3}$sin30
= 2($\frac{\sqrt{3}}{2}$) + $\sqrt{3}$($\frac{1}{2}$) = $\frac{3\sqrt{3}}{2}$

y' = $_A cos\theta + x_A sin\theta$ = -$\sqrt{3}$cos(-210) + (-2)sin(-210)
= -$\sqrt{3}$cos150 – 2sin150
= $\sqrt{3}$cos30 – 2sin30 =$\sqrt{3}$ ($\frac{\sqrt{3}}{2}$) -2 ($\frac{1}{2}$) = $\frac{1}{2}$

The image is A'($\frac{3\sqrt{3}}{2}$; $\frac{1}{2}$) **[6]**

QUESTION 2

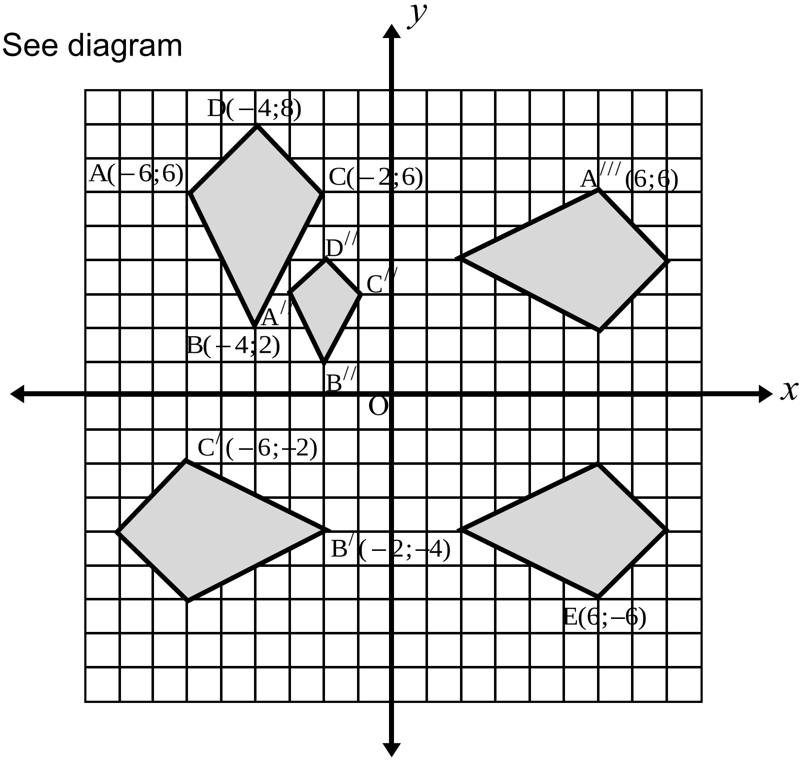
2.1 Reflection in y=-x: (x; y) → (-y; x) (4)
P'(5; 0) and R'(-5;0)

2.2 P''(5cos45;5sin45) ∴ P''(3.54; 3.54) (4)

2.3 S'''(-5cos60; -5sin60)
S'''(-5($\frac{1}{2}$); $-5(\frac{\sqrt{3}}{2})$) ∴ S'''($\frac{-5}{2}$; $\frac{-5\sqrt{3}}{2}$) (4)
[12]

QUESTION 3

3.1 & 3.2 See diagram



(1)

(2)

3.3 $(x\,;y) \rightarrow (-y\,;x)$ (1)

3.4 See diagram (2)

3.5 $\dfrac{\text{Area ABCD}}{\text{Area A}^{//}\text{B}^{//}\text{C}^{//}\text{D}^{//}} = \dfrac{1}{k^2} = \dfrac{1}{(\frac{1}{2})^2} = 4$

(2)

3.6 See diagram (2)

3.7 See diagram

(2)

3.8

$(x\,;y) \rightarrow (x\,;-y)$

$(x\,;-y) \rightarrow (x+7\,;-y)$

$(x+7\,;-y) \rightarrow (2(x+7)\,;-2y)$

$\therefore (x\,;y) \rightarrow (2x+14\,;-2y)$

(4)

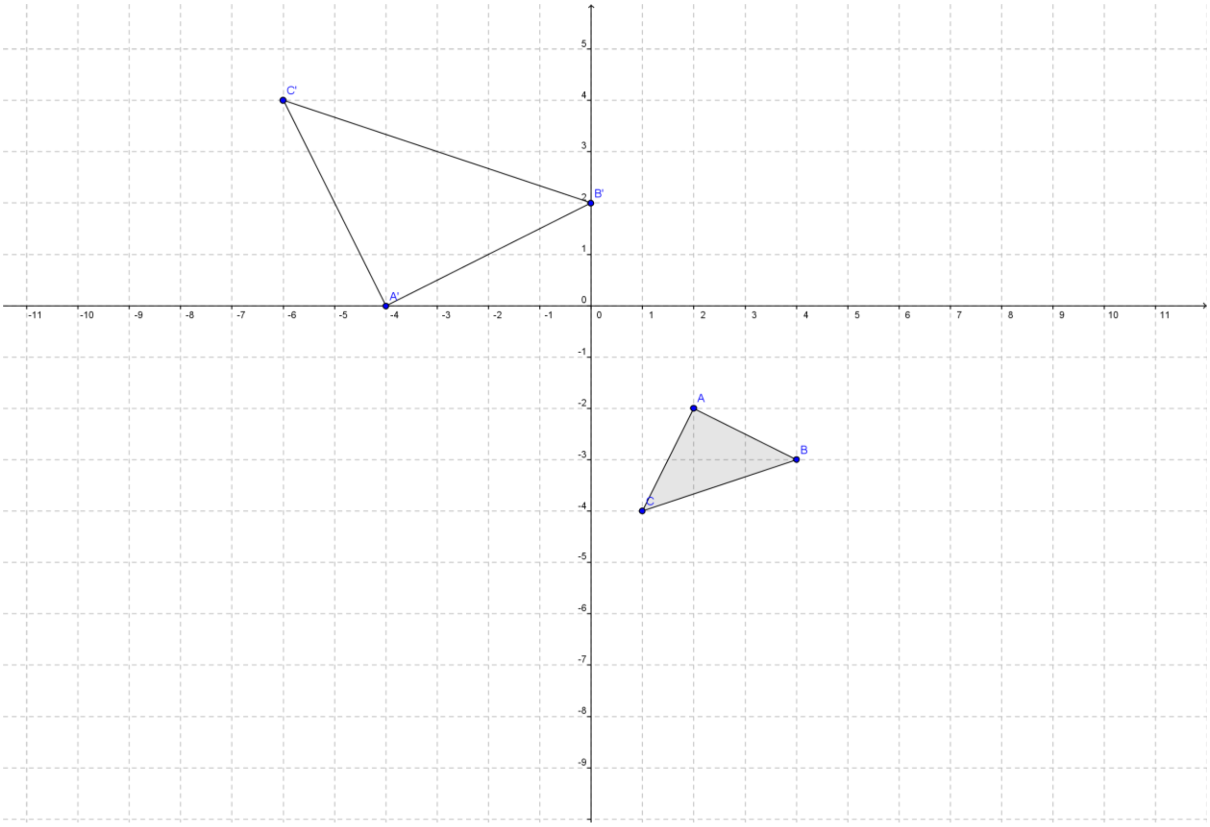
[16]

QUESTION 4

$x^2 + \ ^2 - 2x - 4y$ =4
$(x-1)^2$+ $(y-2)^2$ = 9
∴ the centre is (1;2) and the radius = 3
Rule for rotation through 90°anti-clockwise about the origin; (x; y) → (-y; x)
∴ The centre of the image is (-2;1) The image is then enlarged by a factor of 2
∴ the centre is (-4;2) and the radius = 6.
New Eqn: $(x+4)^2$+ $(y-2)^2$ = 36 **[6]**

QUESTION 5

5.1



Note:

- If the candidate only draws the correct triangle with labels, **full marks**
- If they plot the points correctly and do not draw the triangle, **max 5 / 6 marks**
- In the 3 sketches, if one vertex of the three is wrong, then 1 / 2 marks for the incorrect sketch, then CA applies.
- If they write down the points and do not plot the points and draw the triangle **max 3 / 6 marks**
- If the vertices are correct but not labelled and the points are joined **max 5 / 6 marks**
- If the vertices are correct, not labelled and not joined **max 4 / 6 marks**
- If a candidate finds a formula first and gets it wrong

Max 1 mark for the formula

Max 2 marks for the calculation of A'B'C' coordinates (CA)

1 mark for plotting 3 vertices

1 mark for completing the triangle and labelling

5.2 $(x; y) \rightarrow (x; -y) \rightarrow (x-4; -y-2) \rightarrow (2x-8; -2y-4)$ (If the candidate gives the answer only-award full marks)

5.3 (Please note: there are several different ways to do this question)

M*AC*=2 and M*AB*= -0.5

Therefore CÂB=90 (M*AC x* M*AB=-1)*; AB= √5 and AC= √5

Area of ∆ABC = 0.5 (√5)(√5) = 5/2

∴ Area of ∆ A'B'C' = 4 x 5/2 = 10 square units **[14]**

QUESTION 6

6.1 A' (-12; -6) (1)

6.2 x' = $x_A cos\alpha$ - $y_A sin\alpha$

-12cosα – 6sinα = -12

-2cosα – sinα =-2 (1)

y' = $_A cos\alpha$ + $x_A sin\alpha$ = 6cosα – 12 sinα = -6

cosα = 2sinα -1 (2)

substitute (2) into (1)

-2(2sinα -1) – sinα =-2

-4sinα +2 –sinα = -2

-5sinα = -4 sinα =4/5 ……. α =53,13° (6)

[7]

SESSION 17.1

TOPIC: FUNCTIONS

!

Learner Note: You must be able to sketch all graphs from grades 10&11, including their inverses. It is important to understand how to find domain & range, intercepts, turning points, asymptotes and be able to interpret and identify all graphs. Vertical shifts affect the y values and horizontal shifts affect the x values.

Inverse graphs: A reflection in the line y=x will produce an inverse graph.

Remember only one-to-one graphs will have inverse functions on the entire domain, in the case of many-to-one functions the domain must be restricted in order to find an inverse function on that restricted domain. Use the horizontal line test to tell if a function is one-to-one or many-to-one. Use the vertical line test to tell if a graph is a function.

TIPS: To find the equation of the inverse graph: swop x & y values and then make y the subject of the formula.

The inverse of an Exponential graph is a Log graph; you must be able to convert from exponential to Log form and vice-versa.

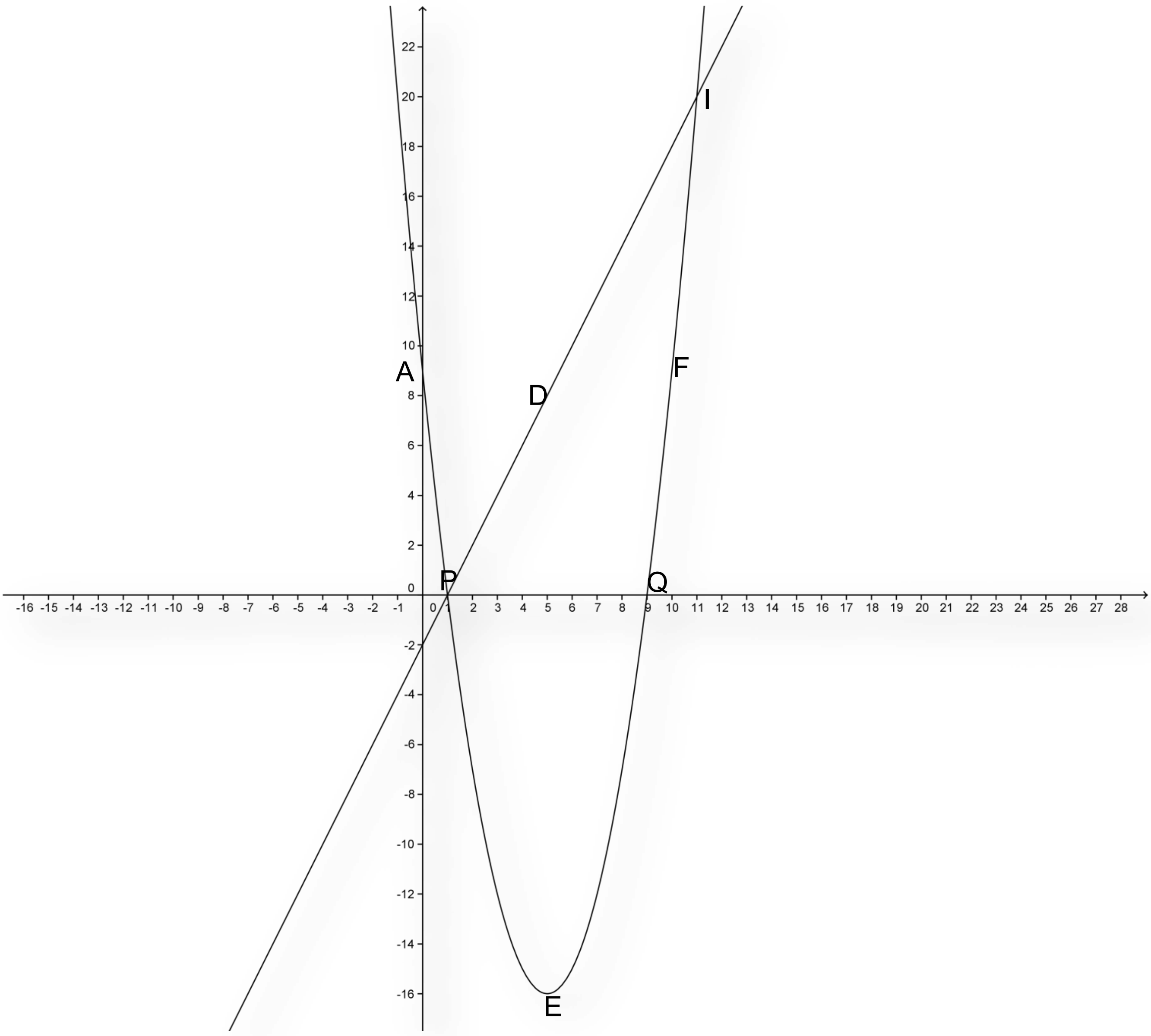
The domain of the original function will become the range of the inverse graph, when reflected in the line y=x, and the range of the original graph will become the domain of the inverse graph.

SECTION A: TYPICAL EXAM QUESTIONS

QUESTION 1

On the following page, the diagram, which is drawn to scale, shows the graphs of $f(x) = x^2 - 10x + 9$ and $(x) = 2x + c$, where c ∈ ℝ.

The graph of *f* intersects the x-axis at P and Q and the y-axis at A. I and P are points of intersection of *f* and *g.* E is the turning point of *f.* F is a point on *f* such that AF is parallel to the x-axis and D is a point on *g* such that DE is parallel to the y-axis.



1.1 Show that the coordinates of P and Q are (1,0) and (9,0) respectively. (2)

1.2 Show that c=-2 (2)

1.3 Determine the lengths of:
1.3.1 AF (2)

1.3.2 DE (4)

1.4 Determine the equation of p(x) if p is the mirror image of *f* in the line y=0 (1)

1.5 Determine the value of x for which the tangent of *f* will be parallel to *g*. (4)
[15]

QUESTION 2

If $f(x) = \frac{2}{x}$ and $g(x) = x^2 - 2$, determine the value of the following:

2.1 p if $g(-1) = p$ (2)

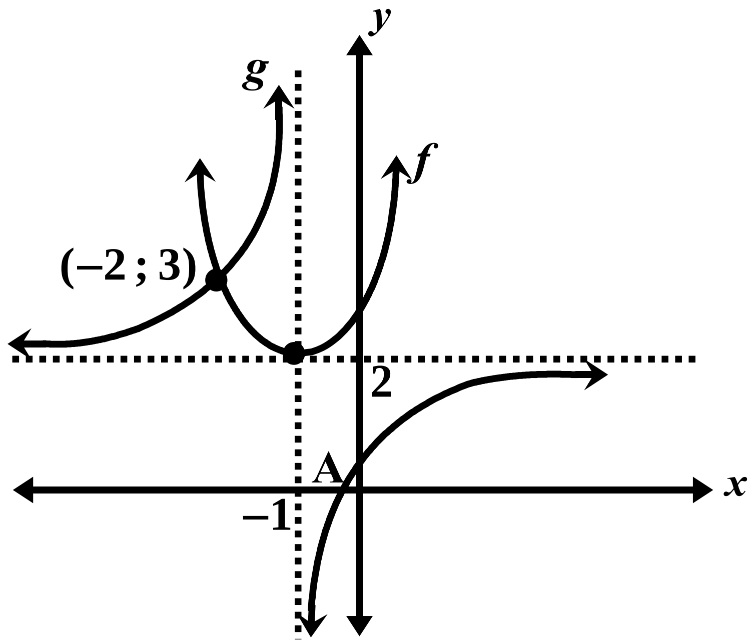
2.2 t if $f(t) = -½$ (2)

2.3 k if $g\left[f(\frac{1}{k})\right] = -1$ (5)

[9]

QUESTION 3

In the diagram below, the graphs of f and g are shown. The graphs intersect at (1;3) . The asymptotes pass through the turning point of the parabola.



3.1 Determine the equation of *f* (3)

3.2 Write down the equations of the asymptotes (2)

3.3 Determine the equation of *g* (3)

3.4 Determine the coordinates of A (3)

3.5 Determine the values of *x* for which $g(x) \leq 0$ (2)

3.6 Determine the equation of the graph formed if *f* is reflected about the *y*-axis (1)

[14]

QUESTION 4

4.1 Sketch the graph of $f(x) = 2^x$ (2)

4.2 Determine the equation of the inverse f^{-1} (2)

4.3 Sketch the graph of the inverse on the same set of axes. (2)

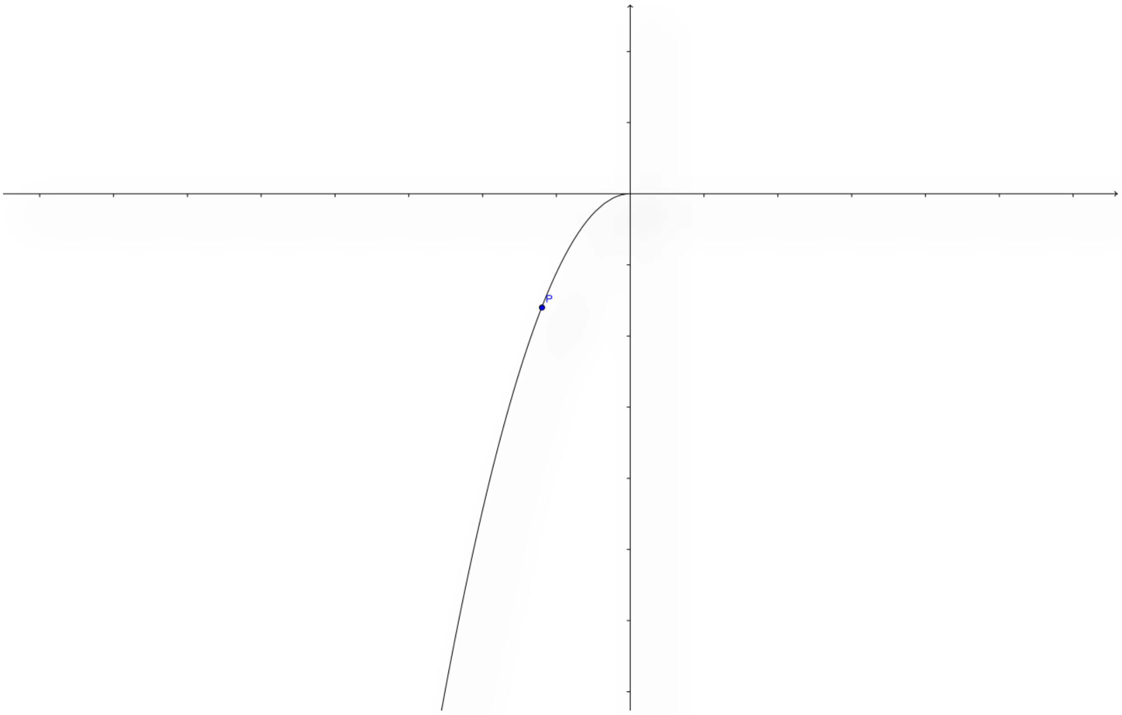
4.4 If the graph of *f* is shifted 1 unit to the right to form the graph *g*, write down the equation of *g*. (1)

4.5 Sketch the graph of *g* on a separate set of axes (2)

[9]

QUESTION 5

The graph of $(x) = ax^2$, $x \leq 0$ is sketched below. The point P(– 6 ; – 8) lies on the graph of *f*.



5.1 Calculate the value of *a* (2)

5.2 Determine the equation of f^{-1} , in the form *y* = … (3)

5.3 Write down the range of f^{-1} (1)

5.4 Draw the graph of f^{-1} on a set of axes. Indicate the coordinates of a point on the graph different from (0 ; 0) (2)

5.5 The graph of *f* is reflected across the line *y* = *x* and thereafter it is reflected across the *x*-axis. Determine the equation of the new function in the form *y* = (3)

[11]

QUESTION 6

Consider the function $f(x) = \left(\frac{1}{3}\right)^x$

6.1 Is f an increasing or decreasing function? Give a reason for your answer. (2)

6.2 Determine $f^{-1}(x)$ in the form $y = \ldots$ (2)

6.3 Write down the equation of the asymptote of $f(x) - 5$. (1)

6.4 Describe the transformation from f to g if $g(x) = \log_3 x$ (2)
[7]

SECTION B: HOMEWORK

QUESTION 1

Given f(x) = 2x find a simplified expression for: $f(x) + f\left(\frac{1}{x}\right) + \frac{1}{f(x)} + f^{-1}(x)$ [6]

QUESTION 2

Given that a function f satisfies the following conditions :
f(0) =2 , f(-2) = 0 , f'(-1) = 0 and f'(x) > 0 for x $\neq$ 0
Draw a rough sketch of the graph f. [5]

QUESTION 3

Consider the function $f(x) = (x+1)^2 - 4$

3.1 Draw a neat sketch graph indicating the coordinates of the intercepts with the axes, the coordinates of the turning point and the equation of the axis of symmetry. (6)

3.1.1 Give the range. (2)
[8]

QUESTION 4

Given: f(x) = a^x passing through the point (2; ¼) ; g(x) = $4x^2$

4.1 Prove that a = ½ (2)

4.2 Determine the equation of y = f^{-1} (x) in the form y = …. (2)

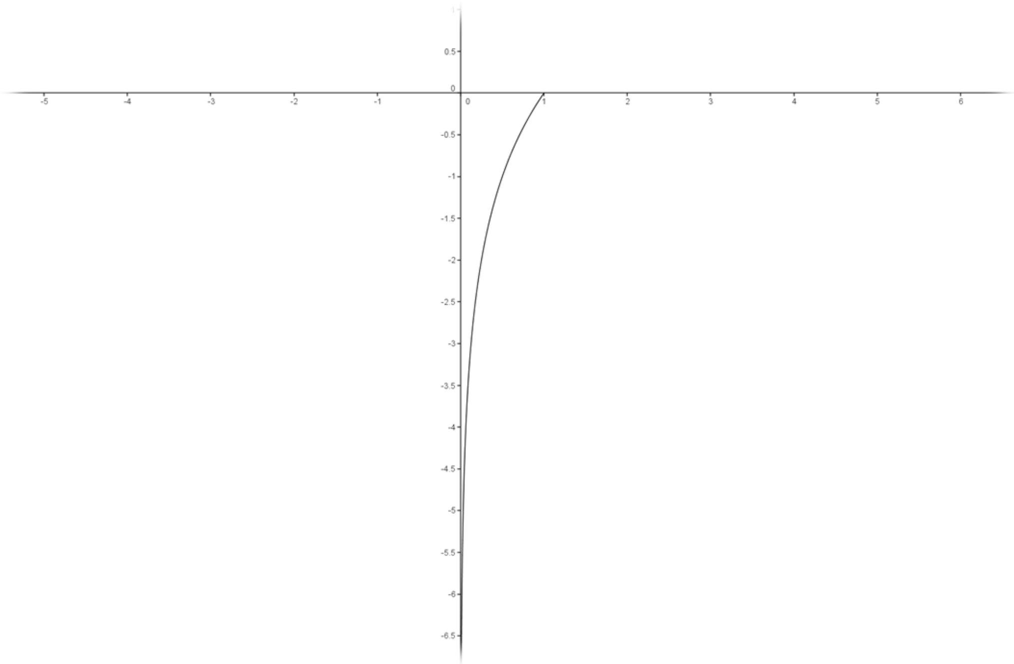
4.3 Determine the equation of y = h(x) where h(x) is the reflection of f(x) about the x-axis (1)

4.4 Determine the equation of the inverse of g in the form y = (2)

4.5 How must the domain of g(x) be restricted so that g^{-1} (x) will be a function? (2)
[9]

QUESTION 5

The graph of g is shown where $\mathrm{g(x)} = \log_a x$ where 0<x<1
(½; -1) is a point on g



5.1 Determine the value of a (2)

5.2 Write down the domain of g(x) (2)

5.3 Write the equation of g^{-1} in the form $g^{-1}(x) =$ ….and state the domain (2)
[6]

SECTION C: SOLUTIONS TO SECTION A

QUESTION 1

1.1 x intercepts: P(1; 0) and Q(9; 0) (2)

1.2 graph g intersects f at P ∴ (1; 0) is a point on g
$g(x) = 2x + c$ Substitute point P……. 2(1) +c = 0 ∴ c=-2 (2)

1.3

1.3.1 can use symmetry and read off the graph itself OR:
Since AF ∥ x axis point F has co-ordinates (x; 9)
∴ $x^2 - 10x + 9 = 9$
x(x-10) = 0
∴ x=10 or x = 0 ∴ F(10; 9) so AF = 10-0 = 10 units (2)

1.3.2 Turning point of f: $x = -\frac{b}{2a} = -\frac{(-10)}{2} = 5$
f(5)= 25-50+9 = -16 ∴ E(5;-16)
distance from the x axis to the T.P E is 16 units
D(5;y) lies on the line g(x): g(5) = 2(5) -2 = 8
Distance DE= 8 +16 = 24 (4)

1.4 p(x) = -f(x) = $-x^2 + 10x - 9$ (1)

1.5 Parallel lines have equal gradients mg(x) =2
$f'(x) = 2x - 10$ =2
2x=12
∴ x =6 (4)
[15]

QUESTION 2

2.1 g(-1) =p
$(-1)^2 - 2 = -1 = p$ (2)

2.2 f(t) = -½
f(t) = $\frac{2}{t} = -\frac{1}{2}$
-t =4 ∴ t= 4 (2)

2.3 $g\left[f(\frac{1}{k})\right] = -1$
$\left[f\left(\frac{1}{k}\right)\right] = \frac{2}{\frac{1}{k}}$ = 2k
$g(2k) = (2k)^2 - 2 = 4k^2 - 2$
$4k^2 - 2 = -1$
$4k^2 = 1$
$k^2 = \frac{1}{4}$
$\therefore k = \pm\frac{1}{2}$ (5)
[9]

QUESTION 3

3.1
$y = a(x+1)^2 + 2$
$3 = a(-2+1)^2 + 2$
$\therefore 1 = a$
$\therefore f(x) = (x+1)^2 + 2$ (3)

3.2
$x = -1$
$y = 2$ (2)

3.3
$y = \frac{a}{x+1} + 2$
$\therefore 3 = \frac{a}{-2+1} + 2$
$\therefore 3 = -a + 2$ (3)
$\therefore a = -1$
$\therefore g(x) = \frac{-1}{x+1} + 2$

3.4

$0=\frac{-1}{x+1}+2$

$\therefore 0=-1+2x+2$

$\therefore -2x=1$

$\therefore x=-\frac{1}{2}$

$\therefore \mathrm{A}\left(-\frac{1}{2};0\right)$ (3)

3.5

$g(x)\le 0$

$\therefore -1<x\le -\frac{1}{2}$ (3)

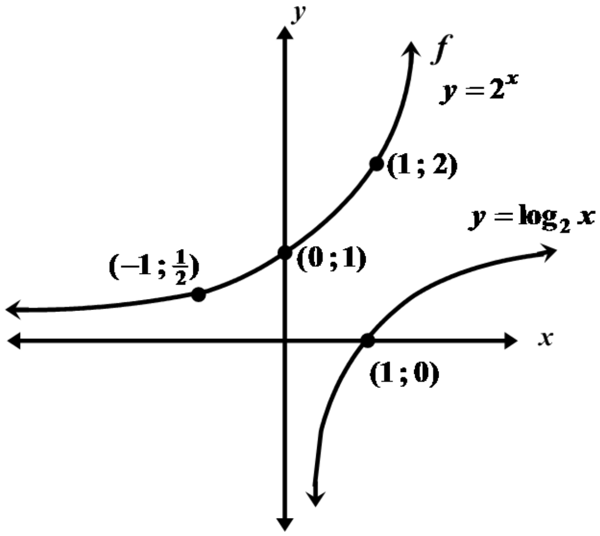
3.6

$y=(x-1)^2+2$ (1)

[14]

QUESTION 4

4.1



(2)

4.2

$y=2^x$

$\therefore x=2^y$

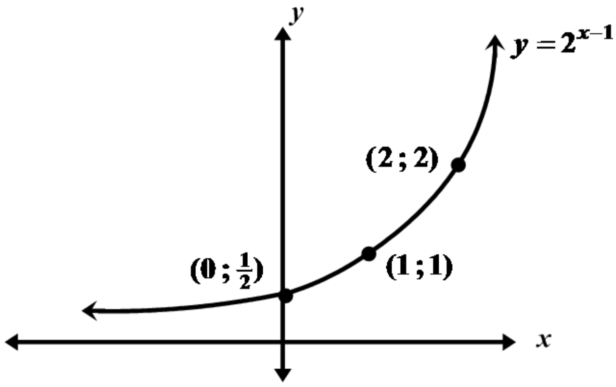
$\therefore y=\log_x 2$

$f^{-1}(x)=\log_x 2$ (2)

4.3 See diagram (2)

4.4 $g(x)=2^{x-1}$ (1)

4.5



(2)
[9]

QUESTION 5

5.1 $f(x) = ax^2$

$a(-6)^2 = -8$

36a = -8

∴ a= $-\frac{2}{9}$ (2)

5.2 $= -\frac{2}{9}x^2$

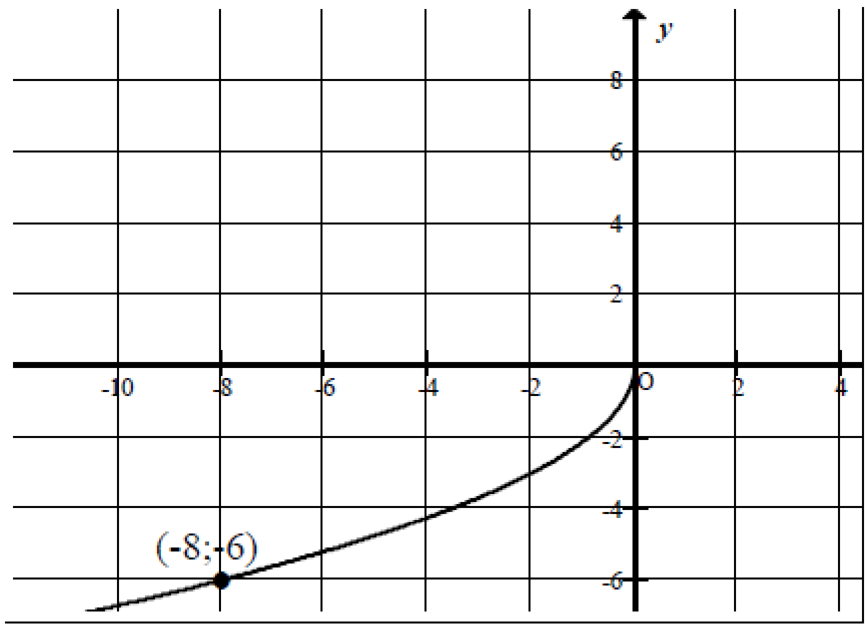
$x = -\frac{2}{9}\ \ ^2$

$^2 = -\frac{9}{2}x$

$= \pm\sqrt{-\frac{9x}{2}}$, since y≤0 $= -\sqrt{-\frac{9x}{2}}$ or y = $-3\sqrt{-\frac{x}{2}}$ (3)

5.3 y≤0 (1)

5.4



(2)

5.5 $= -f^{-1}(x) = -\left(-\sqrt{-\frac{9x}{2}}\right) = \sqrt{-\frac{9x}{2}}$ or $3\sqrt{-\frac{x}{2}}$ (3)

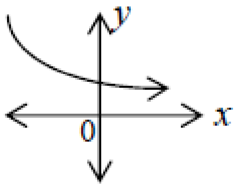
[11]

QUESTION 6

6.1 Decreasing function
Since $0 < a < 1$ OR As x increases, $f(x)$ decreases (2)

6.2
$$f^{-1}: \quad x = \left(\frac{1}{3}\right)^y$$
$$y = \log_{\frac{1}{3}} x$$

OR



OR

$$f^{-1}: \quad x = \left(\frac{1}{3}\right)^y$$
$$y = -\log_3 x$$
(2)

6.3 y=-5 (1)

6.4 Reflection about $y = x$.
Reflection about the x-axis.
OR
Reflection about the y-axis.
Then reflection about the line $y = x$.
OR
Reflection about the line $y = -x$ followed by reflection about the y-axis.
OR
Rotation through 90° in a clockwise direction.
OR
Rotation through 90° in an anti-clockwise direction.
Reflection through the origin. (2)

[7]

SESSION 17.2

TOPIC: CALCULUS

SECTION A: TYPICAL EXAM QUESTIONS

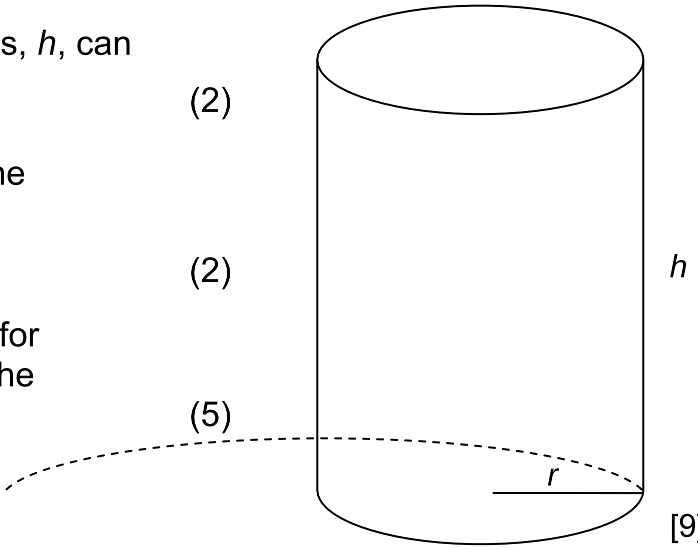
QUESTION 1: 10 minutes

A drinking glass, in the shape of a cylinder, must hold 200 mℓ of liquid when full.

1.1 Show that the height of the glass, *h*, can be expressed as $h = \dfrac{200}{\pi r^2}$. (2)

1.2 Show that the surface area of the glass can be expressed as $S(r) = \pi r^2 + \dfrac{400}{r}$. (2)

1.3 Hence determine the value of *r* for which the total surface area of the glass isa minimum. (5)



[9]

QUESTION 2: 15 minutes

2.1.Differentiate $f(x) = -3x^2$ by first principles. (5)

2.2.If $f(x) = (3x^2 - x)^2$, find $f'(x)$ (4)

2.3.If $y = \frac{2x^2+3x-2}{x+2}$, find $\frac{dy}{dx}$ (3)

2.4.Find $\frac{d}{dx}\left(\frac{4x^3-3x^2}{2\sqrt{x}}\right)$ (4)

[16]

QUESTION 3: 10 minutes

3.1. Evaluate: (3)

$$\lim_{p \to 4} \frac{p^2 - 5p + 4}{4 - p}$$

3.2. Leaving your answer with positive exponents, find $f'(x)$ if: (3)

$$f(x) = 4x^2 - 4x + \frac{1}{x^2} - \frac{1}{2}$$

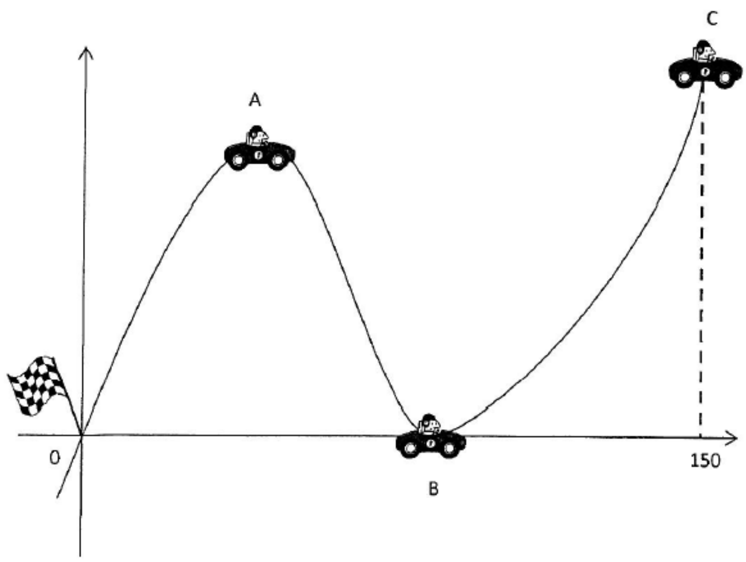
3.3.Evaluate, leaving your answer in surd form, with positive exponents: (4)

$$D_x\left[\frac{5x^2 - 3x}{\sqrt{x}}\right]$$

[10]

QUESTION 4: 15 minutes

Part of a rally track follows the path of a cubic curve. A plain view of this section of the track is shown in the diagram below. At a certain instant, cars A and B are at the turning points of the curve. Car C is at the point where x=150. The race starts at the origin, 0.



The function which describes this part of the track is:

$$f(x) = \frac{1}{900}x^3 - \frac{1}{5}x^2 + 9x$$

4.1. Determine the coordinates of cars A and B (6)

4.2. Find the average gradient between car A and car C. (2)

4.3. Find the equation of the tangent to the track at the starting flag. (2)

4.4. At which point between A and B does a car stop turning to the right and start turning to the left? (3)

[13]

QUESTION 5: 15 minutes

The cross section of a hilly region can be drawn as the graph of $= x^3 - 8x^2 + 16x$, for $0 \leq x \leq 6$, where *x* is measured in kilometres and *y* is the height above sea level in meters.

5.1. Draw the cross section for $0 \leq x \leq 6$. Show all calculations. (7)

5.2. Mark these the peak and the valley on your cross section and calculate the difference in height between the two. (2)

5.3. Determine the gradient of the hill at the point P where x = 1. (2)

5.4. Hence, determine the equation of the tangent to the hill at P. (4)

[15]

SECTION B: ADDITIONAL CONTENT NOTES

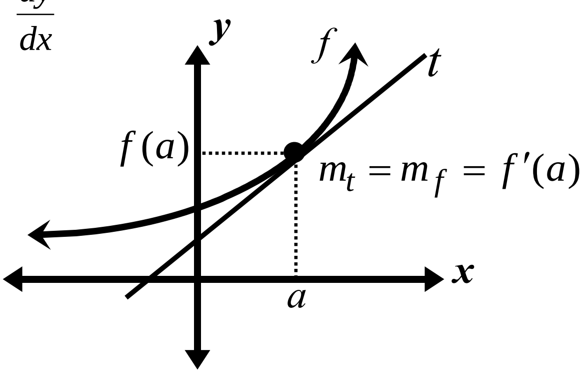
The most important fact in Calculus is that the gradient of the tangent to a curve at a given point is the gradient of the curve at that point.

Other words for gradient are: **rate of change, derivative, slope**

Symbols for gradient are: $f'(x)$ $\quad D_x \quad \frac{dy}{dx}$

$f'(a)$ **is the gradient of** f **at** $x = a$

$f(a)$ **is the** y**-value corresponding to** $x = a$



Average gradient

The average gradient (or average rate of change) of a function f between $x = a$ and $x = b$ and is defined to be the gradient of the line joining the points on the graph of the function. We say that the average gradient of f over the interval is the gradient of the line AB.

Gradient of a curve at a point using first principles

The formula to determine the gradient of a function from first principles is given by the following limit:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

The gradient of a function using the rules of differentiation

You will be required to use the following rules of differentiation to determine the gradient of a function.

Rule 1 If $f(x) = ax^n$, then $f'(x) = a.nx^{n-1}$

Rule 2 If $f(x) = ax$, then $f'(x) = a$

Rule 3 If $f(x) = \text{number}$, then $f'(x) = 0$

Determining the equation of the tangent to a curve at a point

The gradient of the tangent to a curve at a point is the derivative at that point.

The equation is given by $y - y_1 = m(x - x_1)$ where $(x_1 ; y_1)$ is the point of tangency and $m = f'(x_1)$

CALCULUS – GRAPHICAL APPLICATIONS

Rules for sketching the graph of a cubic function

The graph of the form $f(x) = ax^3 + bx^2 + cx + d$ is called a cubic function.
The main concepts involved with these functions are as follows:

Intercepts with the axes:
For the y-intercept, let $x = 0$ and solve for y
For the x-intercepts, let $y = 0$ and solve for x
(you might have to use the factor theorem here)

Stationary points:
Determine $f'(x)$, equate it to zero and solve for x.
Then substitute the x-values of the stationary points into the original equation to obtain the corresponding y-values.
If the function has two stationary points, establish whether they are maximum or minimum turning points.

Points of inflection:
If the cubic function has only one stationary point, this point will be a point of inflection that is also a stationary point.
For points of inflection that are not stationary points, find $f''(x)$, equate it to 0 and solve for x. Alternatively, simply add up the x-coordinates of the turning points and divide by 2 to get the x-coordinate of the point of inflection.

SECTION C: HOMEWORK

QUESTION 1

A builder wishes to construct a steel window frame in the shape of a rectangle with a semi-circular part on top. The radius of the semi-circular part is r metres and the width of the rectangular part is h metres.

1.1 Write down, in terms of h and r

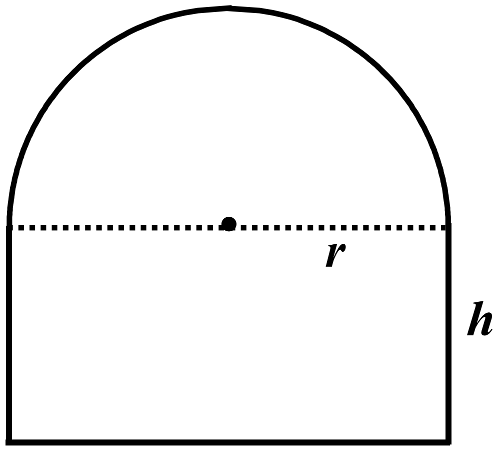
1.1.1 the steel perimeter (P) of the frame. (2)

1.1.2 the area enclosed by the frame. (2)

1.2 The area enclosed by the frame is to be 4 square metres.
Show that the perimeter (P) is

$$\text{P} = \left(\frac{\pi}{2}+2\right)r+\frac{4}{r} \qquad (4)$$

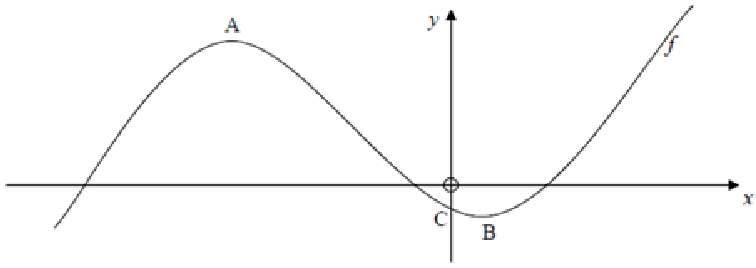
1.3 If the steel for the frame costs R10 per metre, calculate the value of r for which the total cost of the steel will be a minimum. (4)



[12]

QUESTION 2

Refer to the figure. The graph (not drawn to scale) of $f(x) = 4x^3 + 27x^2 - 30x - 1$ is shown with A and B the turning points of the graph.



2.1. Determine the coordinates of A and B. (6)

2.2. Calculate the average gradient of *f* between the points A and B. (2)

2.3. C is the *y*-intercept of the graph. Determine the equation of the tangents to *f* at C. (3)

2.4. Determine the *x*-coordinate of the point on *f* where this tangent cuts the graph again. (3)

[14]

SECTION D: SOLUTIONS AND HINTS TO SECTION A

QUESTION 1

1.1	$V=\pi r^2 h$ $200=\pi r^2 h$ $h=\dfrac{200}{\pi r^2}$	✓ $V=\pi r^2 h$ ✓ $200=\pi r^2 h$ (2)
1.2	$\text{Surface Area}=2\pi rh+\pi r^2$ $S(r)=\pi r^2+\dfrac{200}{\pi r^2}.2\pi r$ $S(r)=\pi r^2+\dfrac{400}{r}$	✓ $S=2\pi rh+\pi r^2$ ✓ $S(r)=\pi r^2+\dfrac{200}{\pi r^2}.2\pi r$ (2)
1.3	$S(r)=\pi r^2+400r^{-1}$ $\dfrac{dS}{dr}=2\pi r-400r^{-2}$ $\text{At minimum}: \dfrac{dS}{dr}=0$ $2\pi r-\dfrac{400}{r^2}=0$ $\pi r^3-200=0$ $r^3=\dfrac{200}{\pi}$ $r=3{,}99\text{ cm}$	✓ $S(r)=\pi r^2+400r^{-1}$ ✓ $\dfrac{dS}{dr}=2\pi r-400r^{-2}$ ✓ $2\pi r-\dfrac{400}{r^2}=0$ ✓ $r^3=\dfrac{200}{\pi}$ ✓ $r=3{,}99\text{ cm}$ (5)

[9]

QUESTION 2

2.1.	$f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $f'(x)=\lim_{h\to 0}\dfrac{-3x^2-6xh-3h^2+3x^2}{h}$ $f'(x)=\lim_{h\to 0}\dfrac{h(-6x-3h)}{h}$ $f'(x)=\lim_{h\to 0}-6x-3h$ $=-6x$	$f(x)=-3x^2$ $f(x+h)=-3(x+h)^2$ $=-3x^2-6xh-3h^2$	✓$-3x^2-6xh-3h^2$ ✓$+3x^2$ ✓ $h(-6x-3h)$ ✓$-6x-3h$ ✓$-6x$ (5)
2.2.	$f(x)=(3x^2-x)^2$ $=9x^4-6x^3+x^2$ $f'(x)=36x^3-18x^2+2x$		✓$9x^4-6x^3+x^2$ ✓$36x^3$ ✓$-18x^2$ ✓$+2x$ (4)

2.3.	$=\frac{2x^2+3x-2}{x+2}$ $=\frac{(2x-1)(x+2)}{x+2}$ $=2x-1$ $\frac{dy}{dx}=2$	✓$(2x-1)(x+2)$ ✓$2x-1$ ✓$\frac{dy}{dx}=2$ (3)
2.4.	$\frac{d}{dx}\left(\frac{4x^3-3x^2}{2\sqrt{x}}\right)$ $=\frac{d}{dx}\left(\frac{4x^3}{2x^{\frac{1}{2}}}-\frac{3x^2}{2x^{\frac{1}{2}}}\right)$ $=\frac{d}{dx}\left(2x^{\frac{5}{2}}-\frac{3}{2}x^{\frac{3}{2}}\right)$ $=5x^{\frac{3}{2}}-\frac{9}{4}x^{\frac{1}{2}}$	✓$2x^{\frac{5}{2}}$ ✓$-\frac{3}{2}x^{\frac{3}{2}}$ ✓$5x^{\frac{3}{2}}$ ✓$-\frac{9}{4}x^{\frac{1}{2}}$ (4)

[16]

QUESTION 3

3.1.	$\lim_{p\to4}\frac{p^2-5p+4}{4-p}$ $=\lim_{p\to4}\frac{(p-4)(p-1)}{-(p-4)}$ $=\lim_{p\to4}-(p-1)$ $=-(4-1)$ $=-3$	✓$(p-4)(p-1)$ ✓$-(p-4)$ ✓-3 (3)
3.2.	$f(x)=4x^2-4x+\frac{1}{x^2}-\frac{1}{2}$ $f(x)=4x^2-4x+x^{-2}-\frac{1}{2}$ $f'(x)=8x-4-\frac{2}{x^3}$	✓$8x$ ✓-4 ✓$-\frac{2}{x^3}$ (3)
3.3.	$D_x\left[\frac{5x^2-3x}{\sqrt{x}}\right]$ $=D_x\left[5x^{\frac{3}{2}}-3x^{\frac{1}{2}}\right]$ $=\frac{15\sqrt{x}}{2}-\frac{3}{2\sqrt{x}}$	✓$5x^{\frac{3}{2}}$ ✓$-3x^{\frac{1}{2}}$ ✓$\frac{15\sqrt{x}}{2}$ ✓$-\frac{3}{2\sqrt{x}}$ (4)

[10]

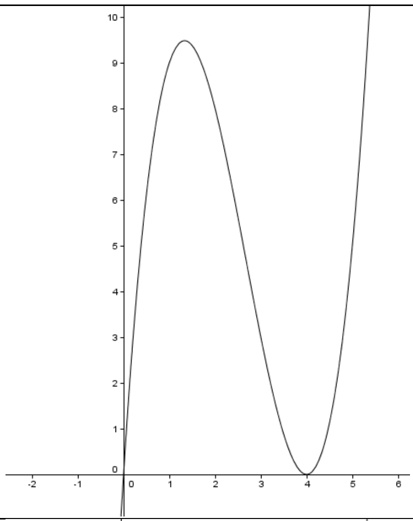
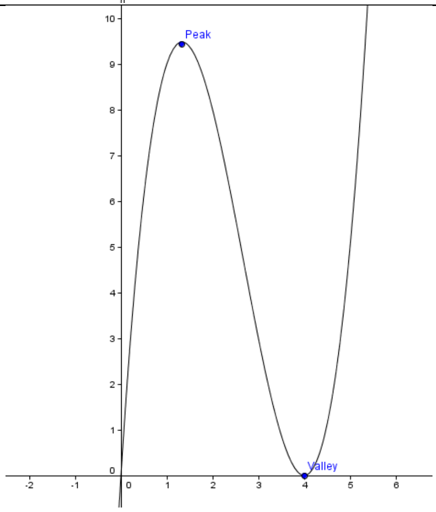
QUESTION 4

4.1.	$f(x)=\frac{1}{900}x^3-\frac{1}{5}x^2+9x$ $f'(x)=\frac{1}{300}x^2-\frac{2}{5}x+9$ $0=\frac{1}{300}x^2-\frac{2}{5}x+9$ $0=x^2-120+2700$ $0=(x-30)(x-90)$ $x=30 \quad or \quad x=90$ $\therefore A(30;12) \quad B(90;0)$	✓$f'(x)=\frac{1}{300}x^2-\frac{2}{5}x+9$ ✓$0=\frac{1}{300}x^2-\frac{2}{5}x+9$ ✓$0=x^2-120+2700$ ✓$0=(x-30)(x-90)$ ✓ $A(30;12)$ ✓$B(90;0)$ (6)
4.2.	$C(150;600)$ $m_{AC}=\frac{600-120}{150-30}$ $m_{AC}=4$	✓$m_{AC}=\frac{600-120}{150-30}$ ✓$m_{AC}=4$ (2)
4.3.	$f'(x)=\frac{1}{300}(0)^2-\frac{2}{5}(0)+9$ $m=9$ $\therefore \quad =9x$	✓$m=9$ ✓$\therefore \quad =9x$ (2)
4.4.	$f''(x)=\frac{1}{150}x-\frac{2}{5}$ $0=\frac{1}{150}x-\frac{2}{5}$ $0=x-60$ $x=60$ $\therefore (60;60)$	✓$\frac{1}{150}x-\frac{2}{5}$ ✓$0=\frac{1}{150}x-\frac{2}{5}$ ✓$(60;60)$ (3)

[13]

QUESTION 5

5.1.	$\quad =x^3-8x^2+16x$ $0=x(x^2-8x+16)$ $0=x(x-4)(x-4)$ $x=0 \quad x=4$ $(0,0)(4;0)$ $3x^2-16x+16=0$ $(3x-4)(x-4)=0$ $x=\frac{4}{3} \qquad x=4$ $\quad =9{,}48 \quad y=0$ $\left(1\frac{1}{3};9{,}48\right)$ $6x-16=0$ $6x=16$ $x=2\frac{2}{3}$ $\quad =4{,}74$ $\left(2\frac{2}{3};4{,}74\right)$	*x*-intercepts: ✓$(0,0)$ ✓$(4;0)$ Turning points: ✓$(4;0)$ ✓$\left(1\frac{1}{3};9{,}48\right)$ Inflection: ✓$\left(2\frac{2}{3};4{,}74\right)$ ✓✓plotted function (7)

		
5.2.	 Difference in height = 9,48km	✓marking the valley and peak on function ✓9,48km (2)
5.3.	$m = 3x^2 - 16 + 16$ $m = 3(1)^2 - 16(1) + 16$ $m = 3$	✓$m = 3x^2 - 16 + 16$ ✓$m = 3$ (2)
5.4.	$\quad = (1)^3 - 8(1)^2 + 16 = 9$ $(1;9) \quad m = 3$ $\therefore \quad - 9 = 3(x-1)$ $\quad = 3x + 6$	✓$(1;9)$ ✓ $\quad - 9 = 3(x-1)$ ✓$3x$ ✓$+6$ (4)

[15]

SESSION 18 .1

TOPIC: LINEAR PROGRAMMING

SECTION A: TYPICAL EXAM QUESTIONS

QUESTION 1

Magical Homes, a home decor company, must produce at least 90 bedroom lamps per week. Not more than 18 people can be employed. An artisan, who earns R600 per week, can produce 9 lamps per week, while an apprentice, who earns R300 per week, only 5 lamps per week. At least one apprentice must be employed for every five artisans. The ratio of apprentices to artisans must not exceed 1:2. Assume that the number of artisans is x and the number of apprentices is y.

1.1 Write down the inequalities. (4)

1.2 Sketch the feasible region. (5)

1.3 How many artisans and apprentices should be employed in order to minimise the wages paid? (7)

[16]

QUESTION 2

TV Mania is a company manufacturing television sets. The company produces two types of television sets: TX and TY. At least one set of both must be manufactured per week. At most 6 sets of TX and 5 sets of TY can be manufactured per week. It takes five hours to manufacture a TX set and 4 hours to manufacture a TY set. A working week is 40 hours. Twice as many workers are required to manufacture TX sets than TY sets, while at least 8 workers are constantly occupied in the manufacturing process.

1.1 Write down the constraints if the company produces x TX sets and y TY sets. (5)

1.2 Represent the system of constraints on graph paper. Indicate the whole number points in the feasible region. (5)

1.3 The company makes a profit of R500 on TX and R250 on TY.

1.3.1 Determine the number of each television set to be manufactured weekly if the company wants to maximise profits. (2)

1.3.2 What number of each set will yield a minimum profit per week? (2)

[14]

SECTION B: ADDITIONAL CONTENT NOTES

Linear programming questions come in two types.

- In the first type, the question will have a graph of a certain situation from which we will answer questions in interpreting the graph. In this case we will normally be asked to give the formula for the inequalities. Normally the feasible region will be given on the graph.
- In the second type, we will be required to draw a graph which describes the situation we are given (word problem). Here we will plot the inequalities and then show the feasible region on the graph. The examples above are of this type.

Steps (for sketching the graph and optimising)

1. Read through the question to identify the information for the inequalities
2. Create your inequalities from the given information (get your less than and greater than signs he right way)
3. Don't forget your implicit constraints (normally x, y ≥0 and elements of the integers)
4. Decide on a scale for the graph by finding the y-intercept and x intercept of each inequality. (this will give you the max x and the max y value so you can choose a suitable scale)
5. Create your graph and plot the inequalities as straight lines.
6. Feasible region decided by getting y on own in each inequality and showing arrows up or down depending on whether y is less than or greater than)
7. Label your vertices A, B, C, D, etc…
8. Get your profit equation from the question (objective function)
9. Get the gradient of the profit equation in order to create a search line. We do this by getting y on its own in the profit function and identifying the gradient m (coefficient of the x value)
10. Slide the search line up or down to get that vertex which is will yield the max or min profit (objective function). For a min it will be the first point from the bottom and for a max it will be the last point reached - within the feasible region

Translating words into constraints

Master the following before attempting any Linear Programming question.

(a) **The sum of x and y has a minimum value of 2 and a maximum value of 8.**

$x+y\geq 2$ **(The sum of x and y is 2 or more)**

$x+y\leq 8$ **(The sum of x and y is 8 or less)**

(b) **At least 2 of x and at most 3 of y.**

$x\geq 2$ **(x can be 2 or more)**

$y\leq 3$ **(y can be 3 or less)**

(c) The ratio of y to x must not be smaller than 3:5

$$y:x \geq 3:5$$
$$\therefore \frac{y}{x} \geq \frac{3}{5}$$
$$\therefore 5y \geq 3x$$

(d) At least 2 of x to 1 of y.

$$x:y \geq 2:1$$
$$\therefore \frac{x}{y} \geq \frac{2}{1}$$
$$\therefore x \geq 2y$$

(e) x may not exceed more than 3 times that of y.

$$x:y \leq 3:1$$
$$\therefore \frac{x}{y} \leq \frac{3}{1}$$
$$\therefore x \leq 3y$$

Sketching inequalities & identifying the feasible regions

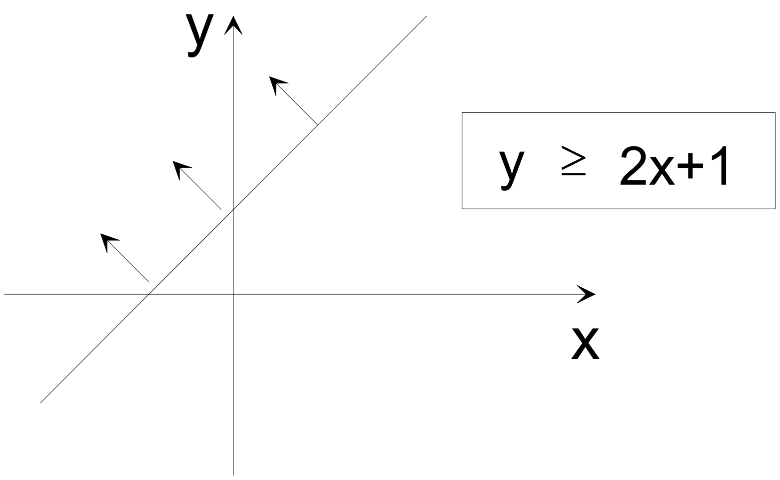
Once we evaluate the inequalities from the given information, we need to sketch them on a graph. We treat them the same as straight lines which you are familiar with. Remember, straight lines have the form $y = mx + c$

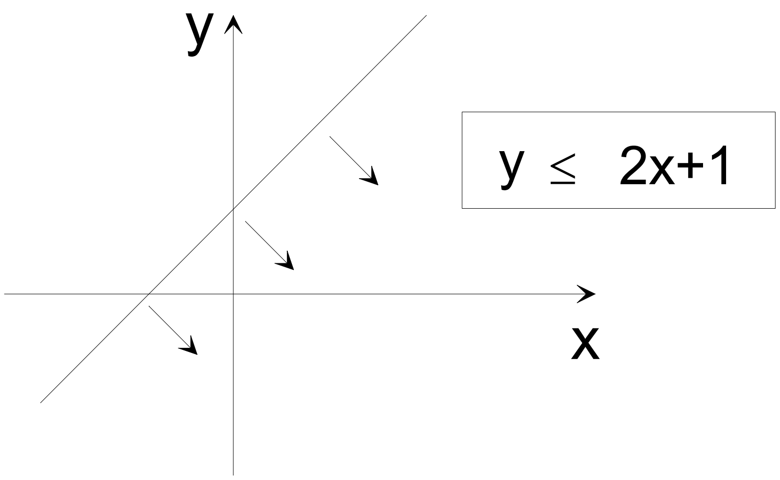
For plotting inequalities we would still need to know the x and y intercepts of the straight lines. But we will always indicate a feasible region for the inequality. This is because the answer to an inequality is a range of values and not a single value.

For example if we change the above equation to an inequality

y≥2x+1

The x intercept is still 1 and the y intercept is still -½. Note that when we get y on its own it will be y≥… this means that the feasible region is above the line. We indicate this by using arrows and this shows us where the feasible region is. The feasible region will be below the line if we have y ≤ 2x+1





Profit/Cost functions

The purpose of a linear programming question is to find those points on the graph that either maximise or minimise a given situation defined by a cost or profit function. Often we try and maximise profit or minimise cost

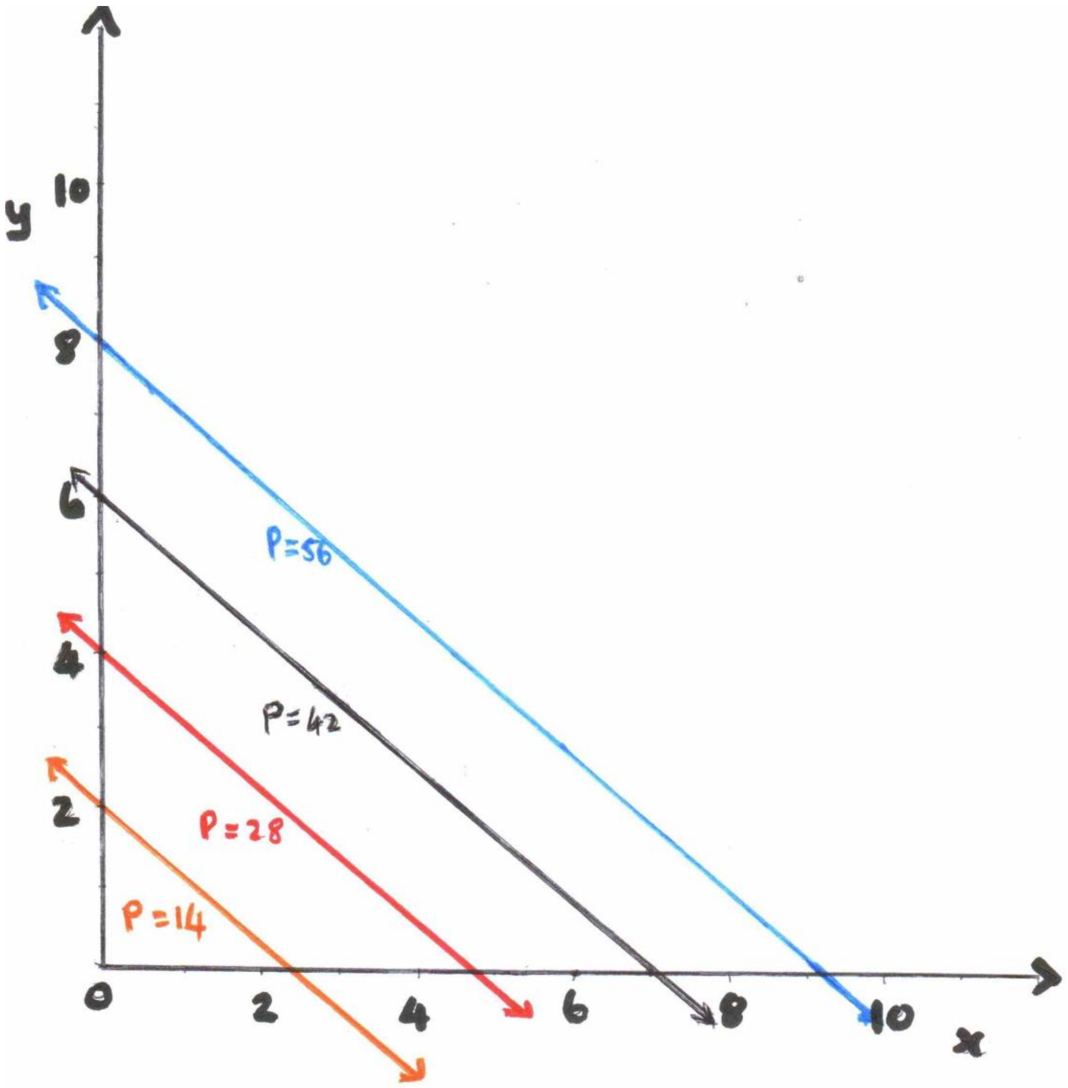
In general, a profit line looks like $P = px + qy$

We use the profit (*P*) to get a search line and we do this by using the gradient of the function.

For example if *P* = 6x+7y

Then y =-6/7x +*P*/7 and so the gradient is -6/7

Plotting this we get the below possible search lines. All the lines below have the same gradient


y
10
8
6
4
2
0
2
4
6
8
10
x
P=56
P=42
P=28
P=14

SECTION C: HOMEWORK

QUESTION 1

A shopkeeper intends buying up to 25 second-hand radios. He has a choice between two types, one without FM for R 30 each and one with FM costing R 40 each.

His total expenditure on the radios must not exceed R 840. If he can sell the first type at a profit of R 10 each and the second type at a profit of R 12 each, determine

1.1 The set of inequalities (3)

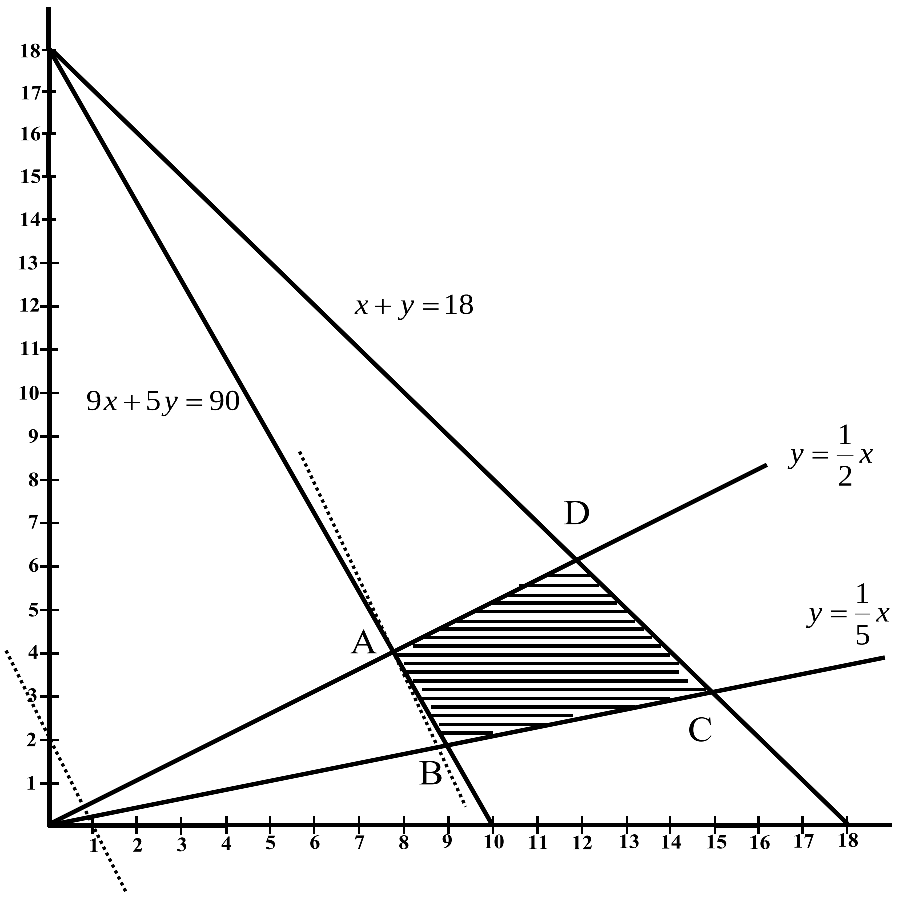
1.2 Sketch the graph and feasible region (5)

1.3 How many of each he should purchase to ensure a maximum profit. (4)
[12]

SECTION D: SOLUTIONS AND HINTS TO SECTION A

QUESTION 1

1.1	Let the number of artisans be x and the number of apprentices be y. $x+y\leq 18$ $9x+5y\geq 90$ $y\leq \frac{1}{2}x$ $y\geq \frac{1}{5}x$	✓ $x+y\leq 18$ ✓ $9x+5y\geq 90$ ✓ $y\leq \frac{1}{2}x$ ✓ $y\geq \frac{1}{5}x$ (4)
1.2	See diagram below	✓ $x+y\leq 18$ ✓ $9x+5y\geq 90$ ✓ $y\leq \frac{1}{2}x$ ✓ $y\geq \frac{1}{5}x$ ✓ Feasible region (5)



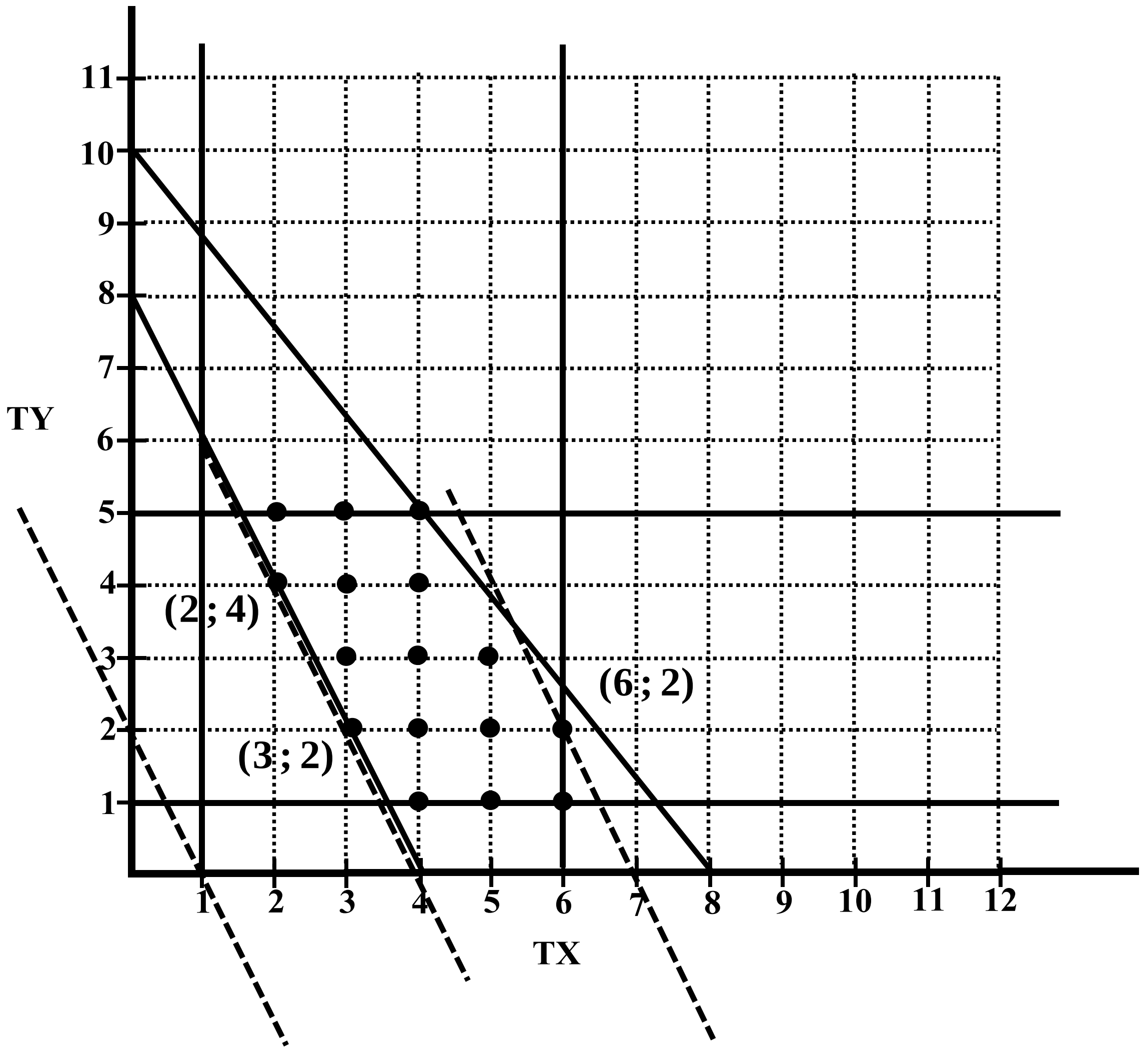
1.3	$C = 600x + 300y$ $\therefore 600x + 300y = C$ $\therefore 300y = -600x + C$ $\therefore y = -2x + \frac{C}{300}$ Point A $\left(7\frac{19}{23} ; 3\frac{21}{23}\right)$ produces minimum cost. Therefore, the company will need to employ: 8 artisans and 4 apprentices	✓ $C = 600x + 300y$ ✓ $y = -2x + \frac{C}{300}$ ✓✓✓✓ $\left(7\frac{19}{23} ; 3\frac{21}{23}\right)$ Note: Simultaneous equations must be used to obtain the value of *x* an *y* ✓ 8 and 4 (7)

[16]

QUESTION 2

2.1	$1 \le x \le 6$ $1 \le y \le 5$ $5x + 4y \le 40$ $2x + y \ge 8$ $x ; y \in N$	✓ $1 \le x \le 6$ ✓ $1 \le y \le 5$ ✓ $5x + 4y \le 40$ ✓ $2x + y \ge 8$ ✓ $x ; y \in N$ (5)

2.2	see diagram below	✓ $1 \leq x \leq 6$ ✓ $1 \leq y \leq 5$ ✓ $5x + 4y \leq 40$ ✓ $2x + y \geq 8$ ✓ $x ; y \in \mathrm{N}$ (5)
2.3.1	$500x + 250y = \mathrm{P}$ $\therefore 250y = -500x + \mathrm{P}$ $\therefore y = -2x + \frac{\mathrm{P}}{250}$ Max at $(6 ; 2)$	✓ $500x + 250y = \mathrm{P}$ ✓ $(6 ; 2)$ (2)
2.3.2	Min at $(2 ; 4)$ and $(3 ; 2)$	✓ $(2 ; 4)$ ✓ $(3 ; 2)$ (2)



[14]

SESSION 18.2

TOPIC: TRIGONOMETRY

Learner Note: You need to know all previous Trigonometry you've have studied as it will be examined as part of your knowledge in the final exam.

If specifically asked to simplify without a calculator marks will not be awarded should a calculator usage be apparent. The trig identities from grade 11 will not be given and must be learnt

i.e. $\sin^2 x + \cos^2 x = 1$ *and* $tan\theta = \frac{sin\theta}{cos\theta}$

SECTION A: TYPICAL EXAM QUESTIONS

QUESTION 1

1.1 Simplify:

$$\frac{\cos(360+x).\cos(90+x).\tan^2(x-180)}{\sin(180-x).\sin(x-360).\tan(-x)}$$ [5]

QUESTION 2

If sin12° = k determine the following in terms of k:

2.1 sin192° (2)

2.2 cos258° (3)

2.3 sin336° (5)

[10]

QUESTION 3

3.1 Prove that: $\frac{sin2A+cos2A+1}{sinA+cosA} = 2cosA$ (6)

3.2 For which values of A is the above identity not defined, for A Є [-180; 180] (3)

[9]

QUESTION 4

Prove without the use of a calculator, that if sin28° = a and cos32° = b, then:

$$b\sqrt{1-a^2} - a\sqrt{1-b^2} = \frac{1}{2}$$

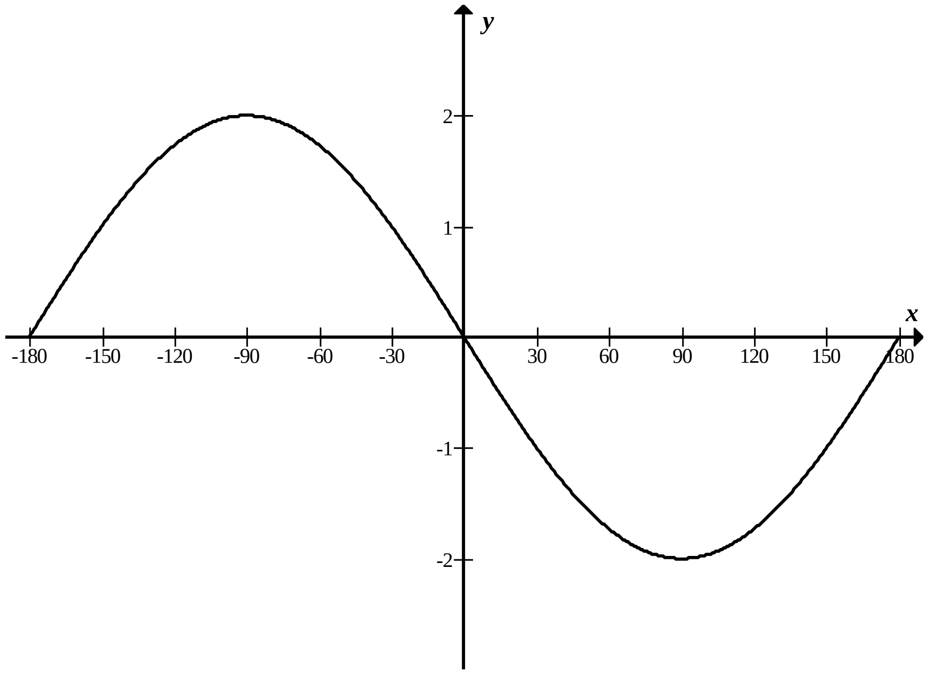
[4]

QUESTION 5

Determine the general solution of: $\sin^2 x + \cos 2x - \cos x = 0$ [7]

QUESTION 6

The graph of f(x)=-2sinx is drawn below



6.1 Write down the period of f. (1)

6.2 Write down the amplitude of h if $h(x) = \frac{f(x)}{4}$ (2)

6.3 Draw the graph of $g(x) = \cos(x - 30°)$ for $x \in [-180° \ ; \ 180°]$ (3)

6.4 Use the graph to determine the number of solutions for $-2\sin x = \cos(x - 30°)$, $x \in [-180° \ ; \ 180°]$ (1)

6.5 For which values of x is g(x) ≥0? (2)

6.6 For which values of x is $f^{/}(x) < 0$ and $g^{/}(x) > 0$? (3)

[12]

SECTION B: HOMEWORK

QUESTION 1

Determine the value of the following without using a calculator:

$$\frac{\sin(-145°).\cos(-215°)}{\sin 510°.\cos 340°}$$ [8]

QUESTION 2

If $\sin 2A = \frac{\sqrt{5}}{3}$, with $2A \in [90° : 270°]$ determine without the use of a calculator:

2.1 cos2A (4)

2.2 tan2A (1)

2.3 sinA (4)

[9]

QUESTION 3

Simplify completely: $\frac{\sin(90+\theta)+\cos(180+\theta)\sin(-\theta)}{\sin 180-\tan 135}$ [5]

QUESTION 4

Prove that for any angle A:

$$\frac{4\sin A.\cos A.\cos 2A.\sin 15}{\sin 2A(\tan 225-2\sin^2 A} = \frac{\sqrt{6}-\sqrt{2}}{2}$$ [6]

QUESTION 5

Determine the general solution of:

$6\cos x - 5 = \frac{4}{\cos x}$; $\cos x \neq 0$ [6]

QUESTION 6

Determine the value of the following without using a calculator:

$\cos^4 375° - \sin^4 345°$ [6]

QUESTION 7

If $\sin 19° = t$, express the following in terms of t. (Make use of a sketch)

7.1 sin79° (7)

7.2 tan71° (3)

[10]

SECTION B: SOLUTIONS TO SECTION A

QUESTION 1

$$\frac{\cos(360+x).\cos(90+x).\tan^2(x-180)}{\sin(180-x).\sin(x-360).\tan(-x)} = \frac{cosx\,(-sinx).\tan^2 x}{sinx.sinx.(-tanx)}$$

$$= \frac{cosx.tanx}{sinx}$$

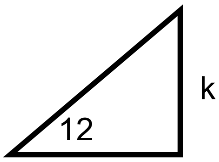
$$= \frac{cosx.sinx}{sinx.cosx} = 1$$ **[6]**

QUESTION 2

2.1 sin192 = sin(180+12) = -sin12 = -k (2)

2.2 cos258 = cos(180+78) = -cos78
= -cos(90-12) = -sin12 = -k (3)

2.3 sin336 = sin(360-24)
= -sin24 = -sin(2× 12) = -2sin12.cos12
= -2k($\sqrt{(1-k^2}$) (5)



[10]

QUESTION 3

3.1 $$\frac{sin2A+cos2A+1}{sinA+cosA} = \frac{2sinAcosA+2cos^2A-1+1}{sinA+cosA}$$

$$= \frac{2cosA(sinA+cosA)}{(sinA+cosA)} = 2cosA$$ (6)

3.2 Identity is undefined if sinA +cosA = 0
sinA = -cosA
tanA = -1 cosA≠0
undefined when A = 135 +k.180 k∈ℤ (3)

[9]

QUESTION 4

$$b\sqrt{1-a^2} - a\sqrt{1-b^2}$$
$$= \cos 32°.\sqrt{1-\sin^2 28°} - \sin 28°\sqrt{1-\cos^2 32°}$$
$$= \cos 32°.\cos 28° - \sin 28°.\sin 32°$$
$$= \cos(32° + 28°)$$
$$= \cos 60°$$
$$= \frac{1}{2}$$

[4]

QUESTION 5

$$\sin^2 x + \cos 2x - \cos x = 0$$
$$\sin^2 x + (\cos^2 x - \sin^2 x) - \cos x = 0$$
$$\cos^2 x - \cos x = 0$$
$$\cos x(\cos x - 1) = 0$$
$$\cos x = 0 \;\; or \;\; \cos x = 1$$
$$x = \pm 90° + k.360° \quad or \;\; x = 0° + k.360° \quad k \in Z$$
$$= k.360°$$

$$(\text{i.e. } x = 90° + k.180° \text{ or } x = k.360° \pm 90°, k \in Z)$$

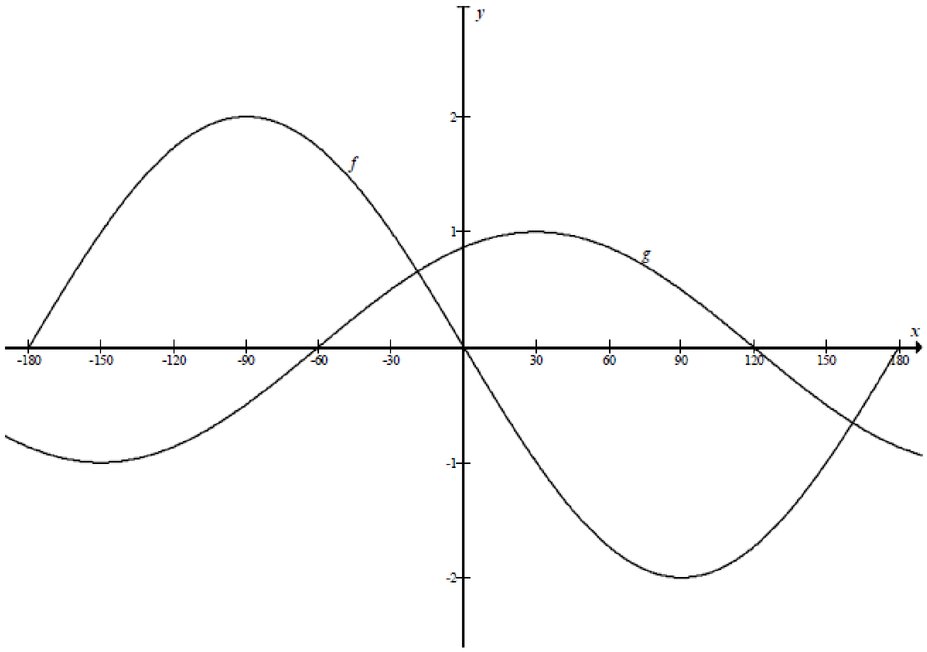
[7]

QUESTION 6

6.1 Period = 360˚ (1)

6.2 Amplitude = ½ (2)

6.3 (3)



6.4 2 solutions (1)

6.5 -60˚≤ x ≤ 120˚ or x€[-60˚; 120˚] (2)

6.6 -90˚< x < 30˚ or x€(-90˚; 30˚) (3)

[12]

SESSION 19.1

TOPIC: DATA HANDLING

Learner Note: Data Handling makes up approximately 20% of paper two. This session is designed in particular to help you understand how to apply what you have learnt in grade 11 to answer questions regarding best fit and distribution of data. It is important that you understand that it is crucial that you are able to interpret a set of data and communicate that.

SECTION A: TYPICAL EXAM QUESTIONS

QUESTION 1

The ages of the final 23 players selected by coach Carlos Perreira to play for Bafana Bafana in the 2010 FIFA World Cup are provided on the following page.

Position	Player	Age
1	Shu-Aib Walters	28
2	Siboniso Gaxa	26
3	Tshepo Masilela	25
4	Aaron Moekoena	29
5	Lucas Thwala	28
6	Macbeth Sibaya	32
7	Lance Davids	25
8	Siphiwe Tshabalala	25
9	Katlego Mphela	25
10	Steven Pienaar	28
11	Teko Modise	27
12	Reneilwe Letsholonyane	28
13	Kagisho Dikgacoi	25
14	Matthew Booth	33
15	Bernard Parker	24
16	Itumeleng Khune	22
17	Surprise Moriri	30
18	Siyabonga Nomvethe	32
19	Anele Ngcongca	22
20	Bongani Khumalo	23
21	Siyabonga Sangweni	28
22	Moeneeb Josephs	30
23	Thanduyise Khuboni	24

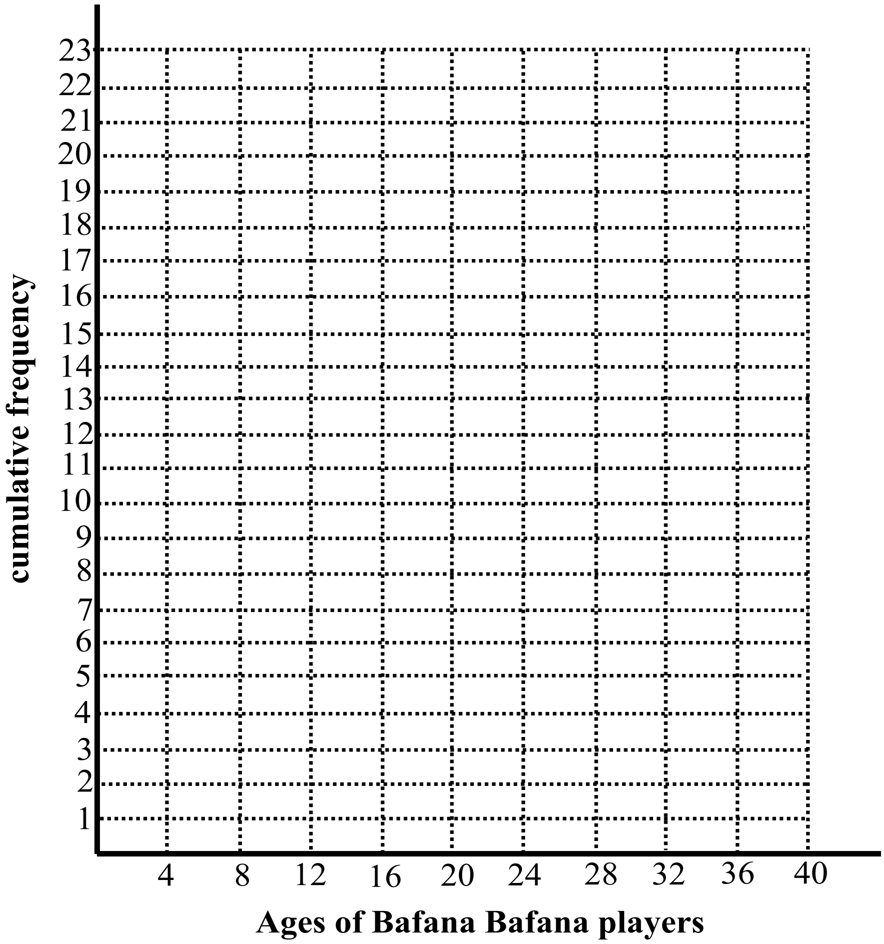
Source: www.2010 Fifa World Cup:final squads – MediaClubSouthAfica.com

The ages of the players are to be grouped into class intervals.

(a) Complete the following table: (2)

Class intervals (ages)	Frequency	Cumulative frequency
$16 \leq x < 20$		
$20 \leq x < 24$		
$24 \leq x < 28$		
$28 \leq x < 32$		
$32 \leq x < 36$		

(b) On the diagram provided below, draw a cumulative frequency curve for this data. (6)



(c) Use your graph to read off approximate values for the quartiles. (3)
[11]

QUESTION 2

(a) Complete the table and then use the table to calculate the standard deviation. (5)

Class intervals	Frequency (f)	Midpoint (m)	$f \times m$	$m - \overline{x}$	$(m-\overline{x})^2$	$f \times (m-\overline{x})^2$
$20 \le x < 24$	3	22				
$24 \le x < 28$	9	26				
$28 \le x < 32$	8	30				
$32 \le x < 36$	3	34				
			$\overline{x} =$			

(b) Hence calculate the standard deviation using the table. (2)

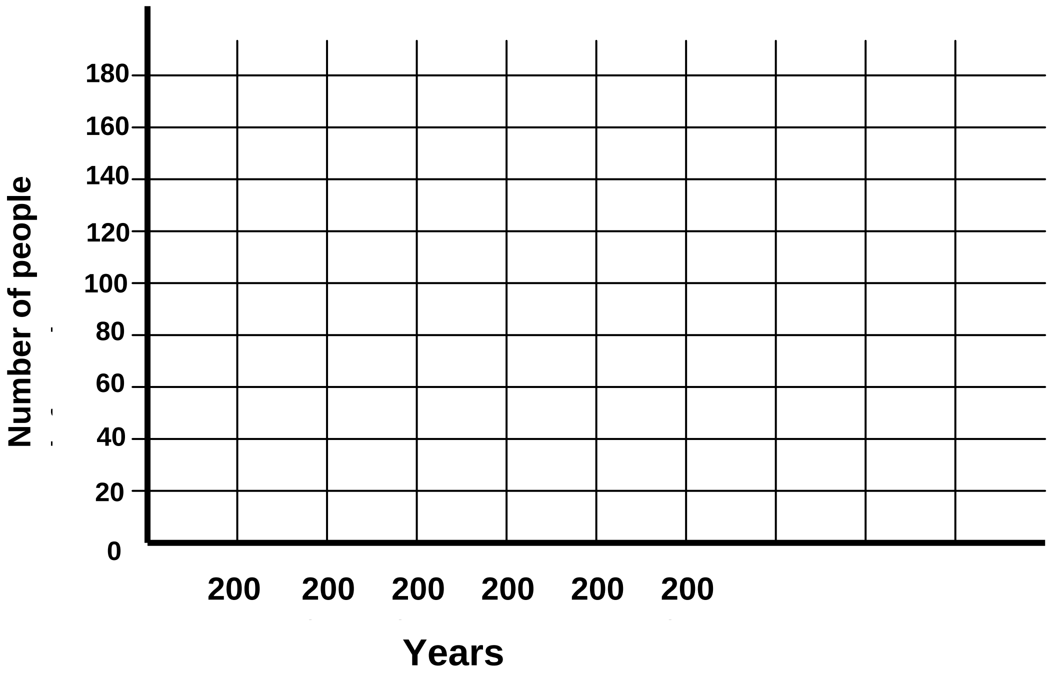
(c) Now use your calculator to verify your answer. (2)
[9]

QUESTION 3

The table below represents the number of people infected with malaria in a certain area from 2001 to 2006:

YEAR	NUMBER OF PEOPLE INFECTED
2001	117
2002	122
2003	130
2004	133
2005	135
2006	137

(a) Draw a scatter plot to represent the above data. Use the diagram provided below.
(2)



(b) Explain whether a linear, quadratic or exponential curve would be a line of best fit for the above-mentioned data. (1)

(c) If the same trend continued, estimate, by using your graph, the number of people who will be infected with malaria in 2008. (1)
[4]

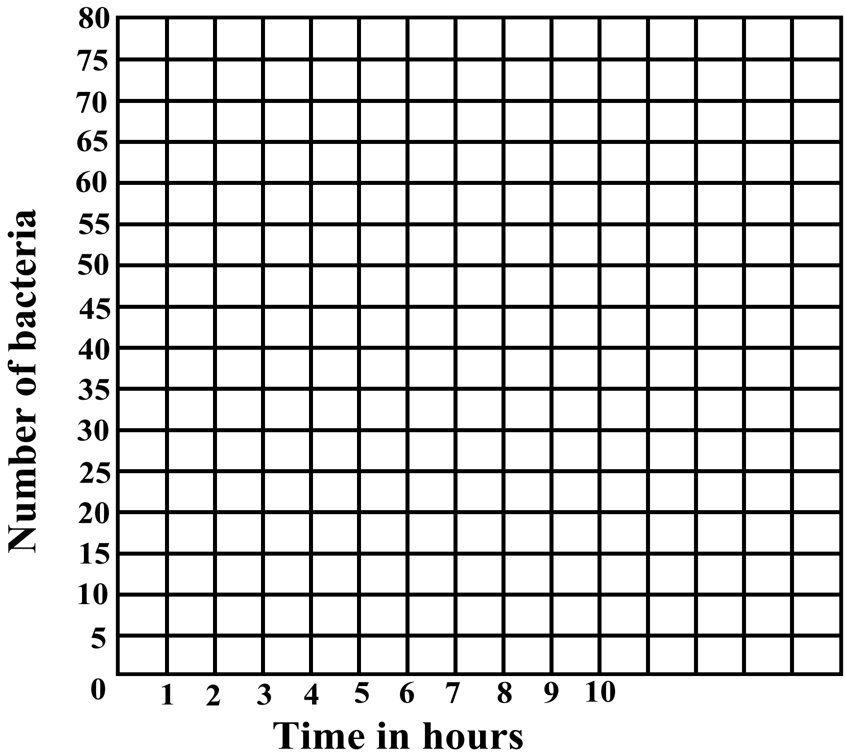
QUESTION 4

A medical researcher recorded the growth in the number of bacteria over a period of 10 hours. The results are recorded in the following table:

Time in hours	0	1	2	3	4	5	6	7	8	9	10
Number of bacteria	5	10	7	13	10	20	30	35	45	65	80

(a) On the diagram provided below, draw a scatter plot to represent this data. (2)

(b) State the type of relationship (linear, quadratic or exponential) that exists between the number of hours and the growth in the number of bacteria. (1)



(3)
[6]

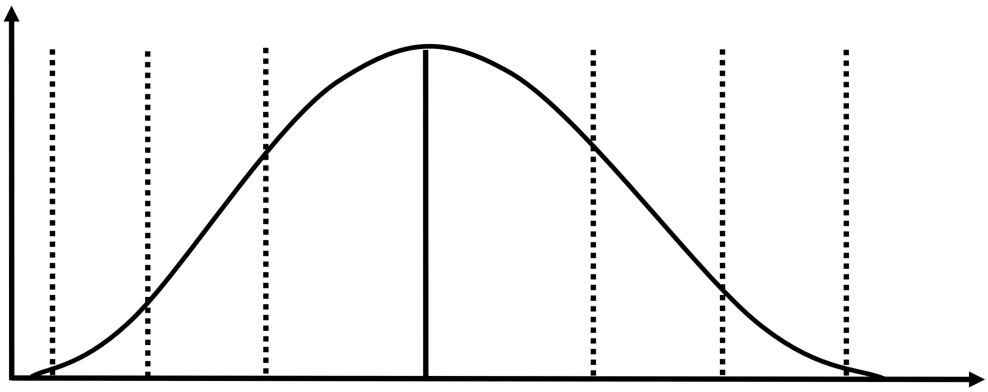
QUESTION 5

The duration of telephone calls made by a receptionist was monitored for a week. The data obtained is represented by the normal distribution curve on the following page. The mean time was 176 seconds with a standard deviation of 30 seconds.

(a) What percentage of calls made was between 146 seconds and 206 seconds in duration? Fill in the necessary information on the graph provided below. (2)

(b) Determine the time interval for the duration of calls for the middle 95% of the data. (2)

(c) What percentage of calls made were in excess of 146 seconds? (2)



[6]

SECTION B – ADDITIONAL CONTENT NOTES

Mean

The mean of a set of data is the average. To get the mean, you add the scores and divide by the number of scores.

Mode

This is the most frequently occurring score.

Quartiles

Quartiles are measures of dispersion around the median, which is a good measure of central tendency. The median divides the data into two halves. The lower and upper quartiles further subdivide the data into quarters.

There are three quartiles:

The Lower Quartile (Q_1): This is the median of the lower half of the values.

The Median (M or Q_2): This is the value that divides the data into halves.

The Upper Quartile(Q_3): This is the median of the upper half of the values.

If there is an odd number of data values in the data set, then the specific quartile will be a value in the data set. If there is an even number of data values in the data set then the specific quartile will not be a value in the data set. A number which will serve as a quartile will need to be inserted into the data set (the average of the two middle numbers).

Range

The range is the difference between the largest and the smallest value in the data set. The bigger the range, the more spread out the data is.

The Inter-quartile range (IQR)

The difference between the lower and upper quartile is called the inter-quartile range.

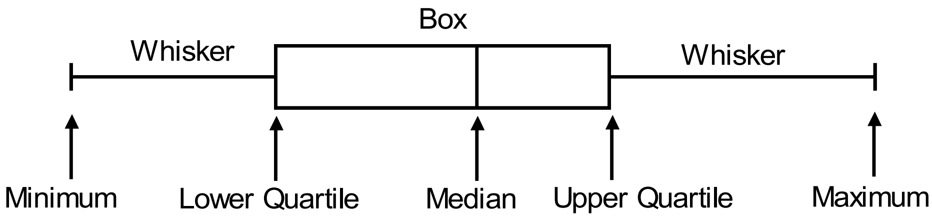
Five Number Summaries

The Five Number Summary uses the following measures of dispersion:

- Minimum: The smallest value in the data
- Lower Quartile: The median of the lower half of the values
- Median: The value that divided the data into halves
- Upper Quartile: The median of the upper half of the values
- Maximum: The largest value in the data

Box and Whisker Plots

A Box and Whisker Plot is a graphical representation of the Five Number Summary.



Standard deviation and variance

Standard deviation and variance are a way of measuring the spread of a set of data. These values also tell us how each value digresses from the mean value. It is important that learners understand what these two concepts are so that they are able to interpret their results and communicate conclusions. Learners need to know how to calculate standard deviation manually using a table as well as their calculators.

The standard deviation (SD) can be determined by using the following formula:

$$\text{SD} = \sqrt{\frac{\sum (x - \bar{x})^2}{n}} \quad variance = \frac{\Sigma(x - \bar{x})^2}{n}$$

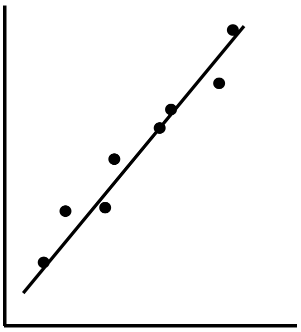
Calculator programmes to calculate standard deviation

CASIO fx-82ES PLUS:
MODE
2 : STAT
1 : 1 – VAR
Enter the data points: push = after each data point
AC
SHIFT
STAT
5: VAR
3 : $x \sigma n$
push = to get standard deviation

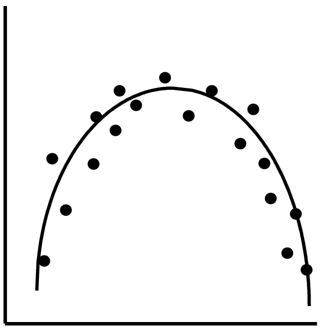
SHARP DAL
MODE 1 =
Enter data points: push STO 2 M+ after each data point
RCL 6 to get standard deviation

Scatter plots and lines and curves of best fit

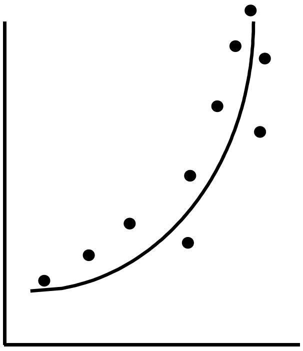
Plotting data on a scatter plot diagram will show trends in the data. Data could follow a linear, quadratic or exponential trend.



Linear



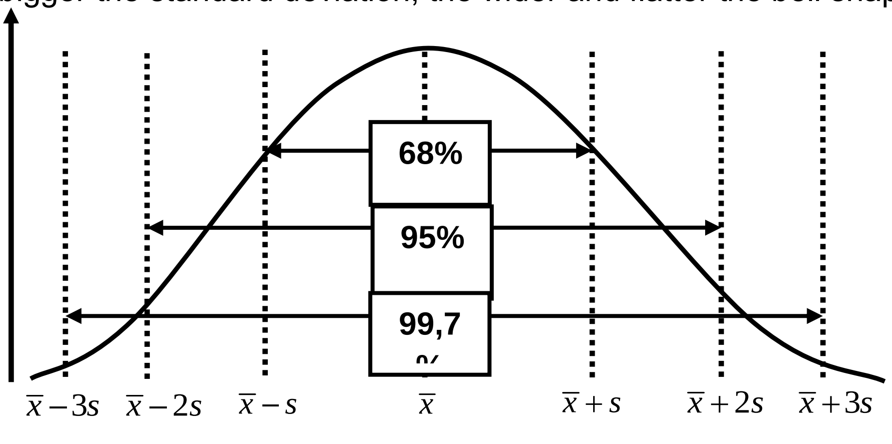
Quadratic



Exponential

The normal distribution curve

1. The mean, median and mode have the same value.
2. An equal number of scores lie on either side of the mean.
3. The majority of scores (99,7%) lie within three standard deviations from the mean, i.e. in the interval $(\overline{x}-3s\,;\overline{x}+3s)$ where $\overline{x}$ represents the mean and s represents the standard deviation.
4. About 95% of scores lie within two standard deviations from the mean, i.e. in the interval $(\overline{x}-2s\,;\overline{x}+2s)$ where $\overline{x}$ represents the mean and s represents the standard deviation.
5. About two-thirds of scores (68%) lie within one standard deviation from the mean, i.e. in the interval $(\overline{x}-s\,;\overline{x}+s)$ where $\overline{x}$ represents the mean and s represents the standard deviation.
6. The smaller the standard deviation, the thinner and taller the bell shape is. The bigger the standard deviation, the wider and flatter the bell shape is.



SECTION C: HOMEWORK

QUESTION 1

The ages of the final 23 players selected by coach Oscar Tabarez to play for Uruguay in the 2010 FIFA World Cup are provided below.

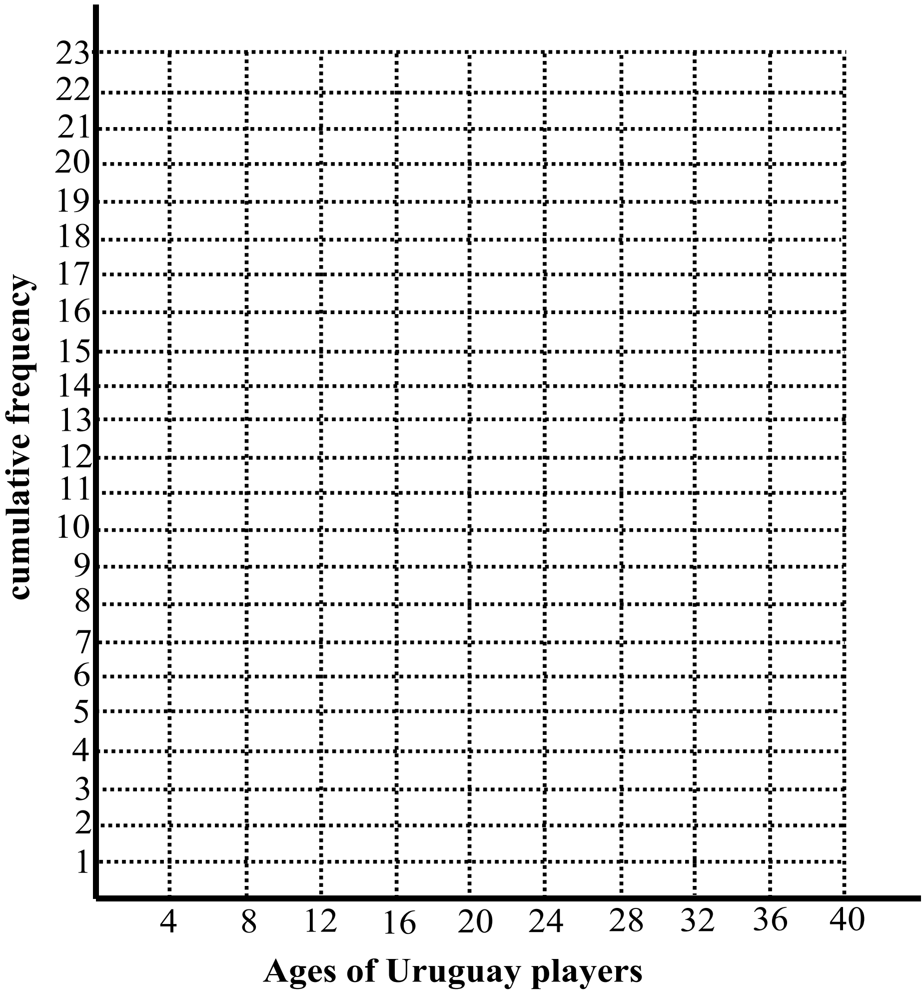
Position	Player	Age
1	Fernando Musiera	23
2	Diego Lugano (captain)	29
3	Diego Godin	24
4	Jorge Fucile	25
5	Walter Gargano	25
6	Andres Scotti	35
7	Edinson Cavani	23
8	Sebastian Eguren	29
9	Luis Suarez	23
10	Diego Forlan	31
11	Alvaro Perreira	25
12	Juan Castillo	32
13	Sebastian Abreu	33
14	Nicolas Lodeira	23
15	Diago Perez	30
16	Maxi Perrreira	26
17	Ignacio Gonzales	28
18	Egidio Arevalo Rios	27
19	Sebastian Fernandes	25
20	Mauricio Victorino	27
21	Alvaro Fernandez	24
22	Martin Caceres	23
23	Martin Silva	27

Source: *www.2010 Fifa World Cup:final squads – MediaClubSouthAfica.com*

(a) Complete the following table: (2)

Class intervals (ages)	Frequency	Cumulative frequency
$16 \leq x < 20$		
$20 \leq x < 24$		
$24 \leq x < 28$		
$28 \leq x < 32$		
$32 \leq x < 36$		

(b) On the diagram provided below, draw an ogive representing the above data. (6)



(c) Use your graph to read off approximate values for the quartiles. (3)
[11]

QUESTION 2

(a) Complete the table and then use the table to calculate the standard deviation. (5)

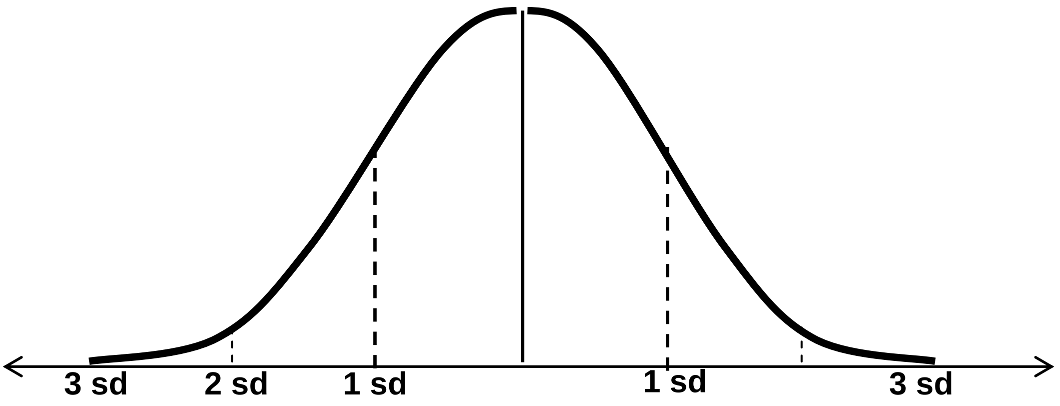
Class intervals	Frequency (f)	Midpoint (m)	$f \times m$	$m - \bar{x}$	$(m - \bar{x})^2$	$f \times (m - \bar{x})^2$
$20 \leq x < 24$	3	22				
$24 \leq x < 28$	9	26				
$28 \leq x < 32$	8	30				
$32 \leq x < 36$	3	34				
			$\bar{x} =$			

(b) Hence calculate the standard deviation using the table. (2)

(c) Now use your calculator to verify your answer. (2)
[9]

QUESTION 3

After protracted union protests, a company analysed its salary structure for employees. They found that the salaries were symmetrically distributed with a mean of R8 850 per month and a standard deviation of R2 950 per month. Research indicated that if the monthly salary was below R3 000, the employee would not maintain an acceptable quality of life.



(a) Estimate the percentage of employees who will struggle to maintain an acceptable quality of life. (2)

(b) Estimate the percentage of employees who earn more than R11 800 per month. (1)

(c) Do you think that the company has a fair salary structure? Use the given data to motivate your answer. (1)

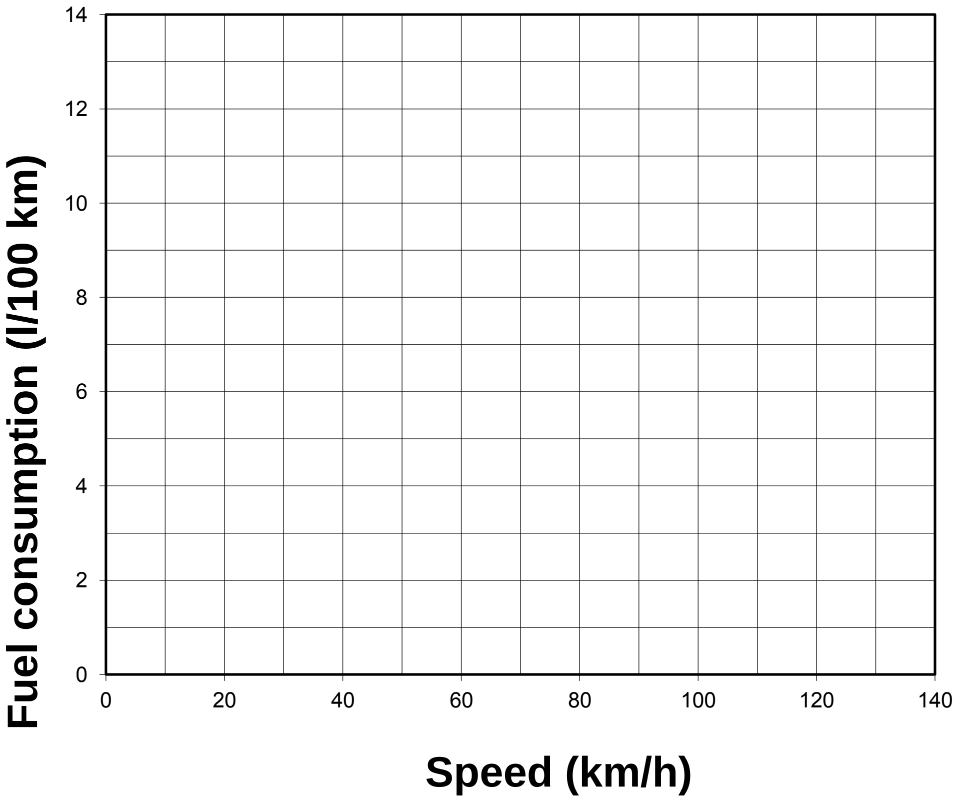
[4]

QUESTION 4

A motor company did research on how the speed of a car affects the fuel consumption of the vehicle. The following data was obtained:

Speed in km/h	60	75	115	85	110	95	120	100	70
Fuel consumption in ℓ/100 km	11,5	10	8,4	9,2	7,8	8,9	8,8	8,6	10,2

(a) Represent the data as a scatter plot on the diagram provided. (3)



(b) Suggest whether a linear, quadratic or exponential function would best fit the data. (1)

(c) What advice can the company give about the driving speed in order to keep the cost of fuel to a minimum? (2)

[6]

SECTION D: SOLUTIONS AND HINTS TO SECTION A

QUESTION 1

(a)

Class intervals (ages)	Frequency ✓	Cumulative frequency ✓
$16 \leq x < 20$	0	0
$20 \leq x < 24$	3	3
$24 \leq x < 28$	9	12
$28 \leq x < 32$	8	20
$32 \leq x < 36$	3	23

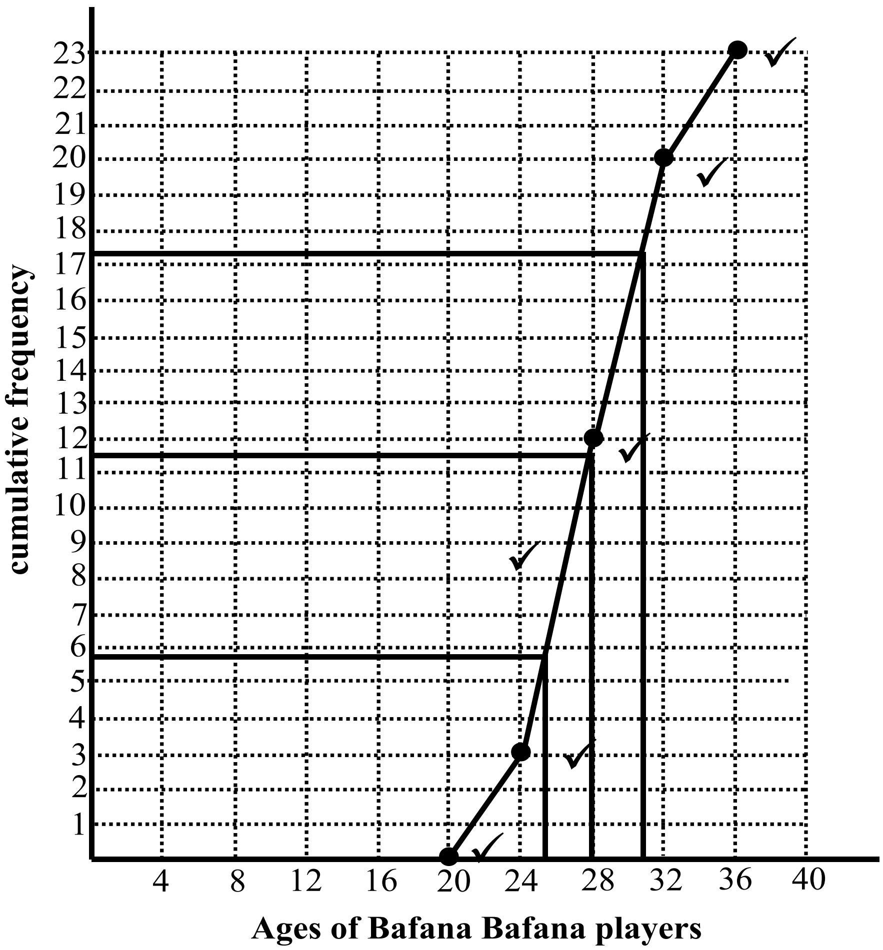
(2)

(b)

Class intervals (ages)	Frequency	Cumulative frequency	Graph points
$16 \leq x < 20$	0	0	$(20;0)$
$20 \leq x < 24$	3	3	$(24;3)$
$24 \leq x < 28$	9	12	$(28;12)$
$28 \leq x < 32$	8	20	$(32;20)$
$32 \leq x < 36$	3	23	$(36;23)$

Learner Note: Cumulative frequency graphs can also be referred to as ogives!

Learner Note: Write down the co-ordinates. X coordinate is the last number in the interval and the y value is the cumulative frequency



(6)

(c)

Lower quartile $$23\times\frac{1}{4}=5.75$$ (5,75;25) Therefore Q_1 = 25✓ **Median** $$23\times\frac{1}{2}=11.5$$ (11.5;28) Therefore Median = 28✓ **Upper quartile** $$23\times\frac{3}{4}=17.25$$ (17.25;31) Therefore Q_3 = 31 ✓	**Learner Note:** Lower quartile is the 25th percentile thus multiply the cumulative frequency by a quarter and read off the graph to determine the y-value. Similarly the median is the 50th percentile thus multiply the cumulative frequency by a half and the upper quartile is determined by multiplying the cumulative frequency by three quarters (the 75th percentile) (3)

[11]

QUESTION 2

(a)

Class intervals	Frequency (f)	Midpoint (m)	$f\times m$ ✓	$m-\bar{x}$ ✓	$(m-\bar{x})^2$ ✓	$f\times(m-\bar{x})^2$ ✓
$20\leq x<24$	3	22	66	–5,9	34,81	104,43
$24\leq x<28$	9	26	234	–1,9	3,61	32,49
$28\leq x<32$	8	30	240	2,1	4,41	35,28
$32\leq x<36$	3	34	102	6,1	37,21	111,63
			$\bar{x}=\frac{642}{23}=27,9$ ✓			$\sum f\times(m-\bar{x})^2=283,83$

(5)

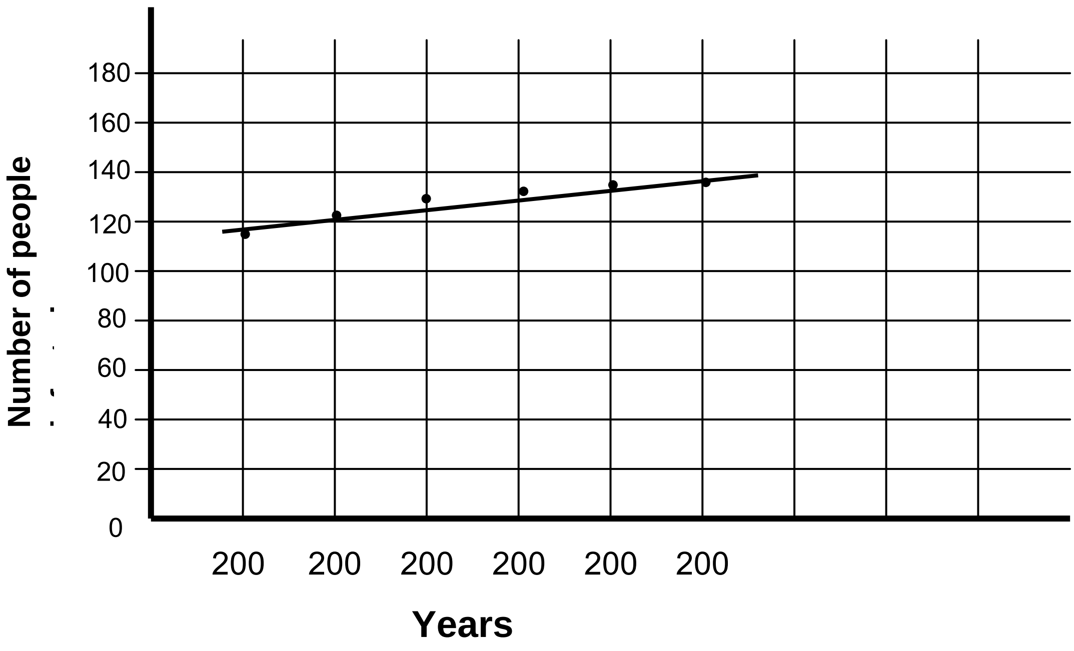
Learner Note: Remember mean can be calculated by multiplying the midpoint of the interval and relative frequency.

(b) $\text{SD}=\sqrt{\frac{\sum f.(x-\bar{x})^2}{23}}=\sqrt{\frac{283,83}{23}}=3,5$	✓✓ (2)

CASIO fx-82ES PLUS: MODE 2 : STAT 1 : 1 – VAR SHIFT SETUP 3: STAT (you need to scroll down to get this function) 1: ON Enter the midpoints: 22= 26= 30= 34= Enter the frequencies: 3= 9= 8= 3= AC SHIFT 1 4: VAR 3 : $x\sigma n$ = The answer will read:3,5 **SHARP DAL:** MODE 1= Enter data: 22 STO 3 M+ 26 STO 9 M+ 30 STO 8 M+ 34 STO 3 M+ RCL 6 to get 3,5	✓✓ (2) **[9]**

QUESTION 3

(a)



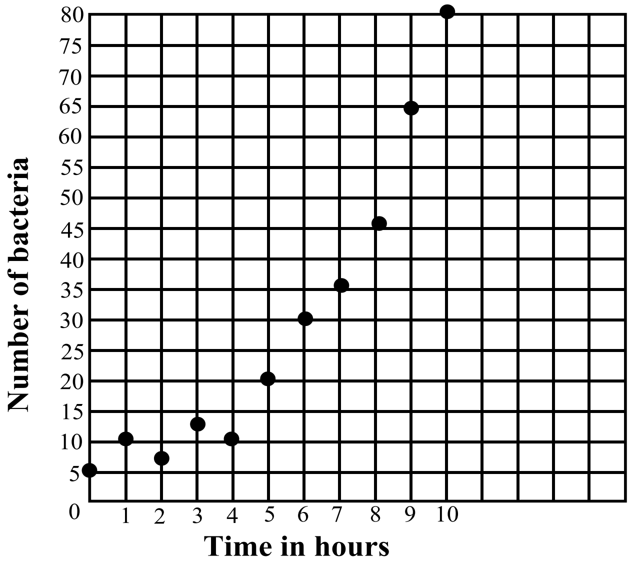
(2)

b. Linear	✓
c. Approx. 145 people	✓

[4]

QUESTION 4

✓✓

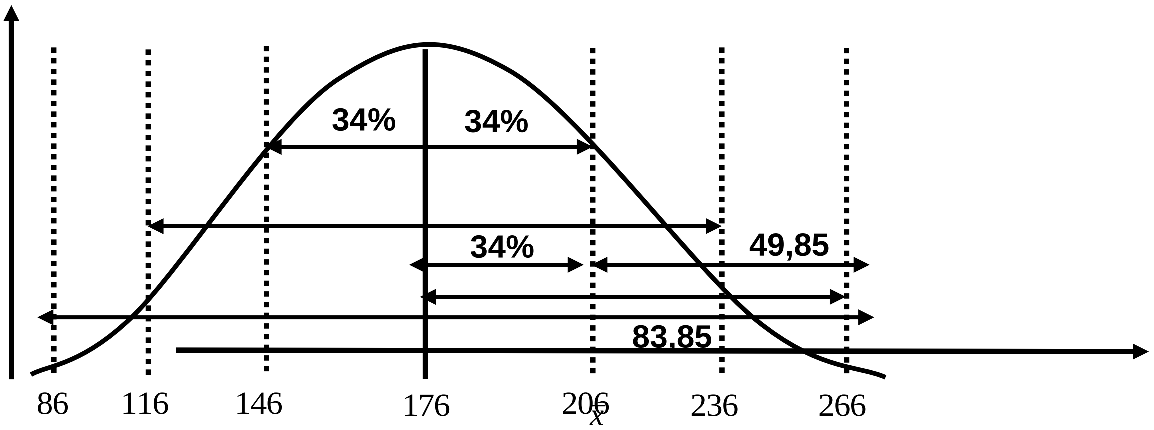


(2)

b. Quadratic or exponential	✓ **Learner Note:** If ever asked to describe or state the relationship, answer by identifying the function, e.g. linear, quardratic, exponential. (1)

[3]

QUESTION 5



One standard deviation interval:

$(\bar{x}-s\,;\bar{x}+s)$

$=(176-30;176+30)$

$=(146;206)$

Two standard deviation intervals:

$(\bar{x}-s\,;\bar{x}+s)$

$=(176-2(30);176+2(30))$

$=(116;236)$

Three standard deviation intervals:

$(\bar{x}-s\,;\bar{x}+s)$

$=(176-3(30);176+3(30))$

$=(86;266)$

a. The interval between 146 seconds and 206 seconds lies between one standard deviation of the mean. For the normal distribution, approximately 68% of the data lies between one standard deviation of the mean.	✓✓
b. The middle 95% of the data for a normal distribution lies between two standard deviations on either side of the mean. The middle 95% of the calls will be between 116 and 236 seconds.	✓✓
c. Approximately 34% of the calls are between 146 and 176 seconds. Another 49, 85% of the calls are in excess of 176 seconds. Therefore, in total, approximately 84% of the calls are in excess of 146 seconds.	✓✓ **[6]**

SESSION 19.2 SELF STUDY

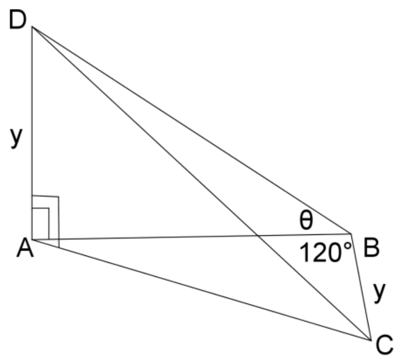
TOPIC: 3D TRIGONOMETRY

Learner Note: Take time to understand all given information, including the diagram. Fill in as much information as possible on the diagram and then work from the triangle with the most information.
Please revise 2D Trigonometry as you will need all the rules learned thus far to solve triangles.

SECTION A: TYPICAL EXAM TYPE QUESTIONS

QUESTION 1

In the figure A,B &C are three points in the horizontal plane, such that ∠ABC = 120˚.
D is a point directly above A
AD = BC =y
The angle of elevation of D from B is θ



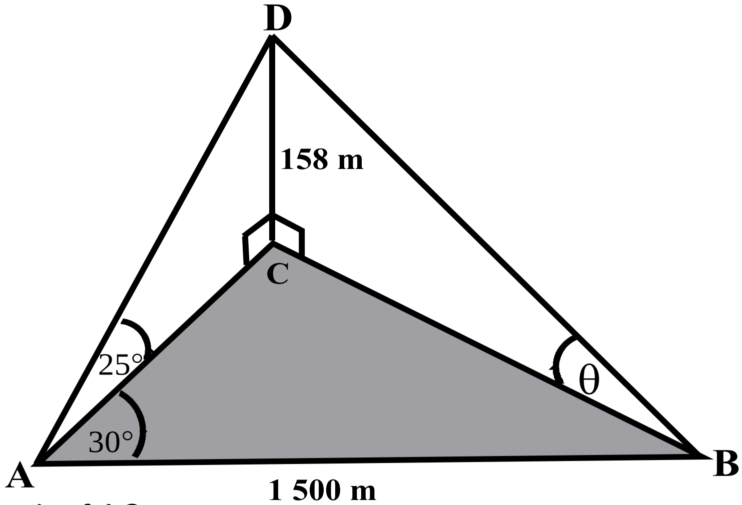
a) Show that AC = $y\sqrt{(\frac{}{\tan^2\theta} + \frac{}{tan\theta} + 1)}$ (7)

b) If y=15 and θ = 22, calculate:

i)AC (2)
ii) ∠ADC (3)
[12]

QUESTION 2

In the diagram below, AB is a straight line 1 500 m long. DC is a vertical tower 158 metres high with C, A and B points in the same horizontal plane. The angles of elevation of D from A and B are $25°$ and θ. $C\hat{A}B = 30°$.



(a) Determine the length of AC. (3)

(b) Find the value of θ. (5)

(c) Calculate the area of ΔABC. (2)

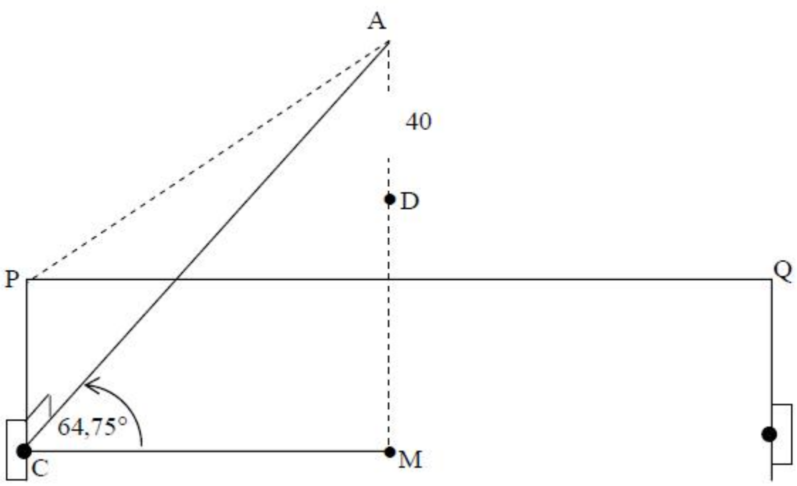
(d) Calculate the size of $A\hat{D}B$ (6)

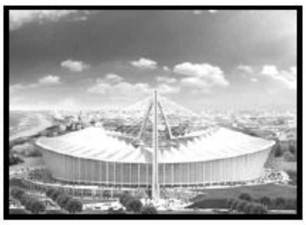
[16]

QUESTION 3 *(DOE Nov 2010 P2)*

The angle of elevation from a point C on the ground, at the centre of the goalpost, to the highest point A of the arc, directly above the centre of the Moses Mabhida soccer stadium, is 64,75°.

The soccer pitch is 100 metres long and 64 metres wide as prescribed by FIFA for world cup stadiums. Also AC ⊥ PC. In the figure below PQ = 100 metres and PC = 32 metres.





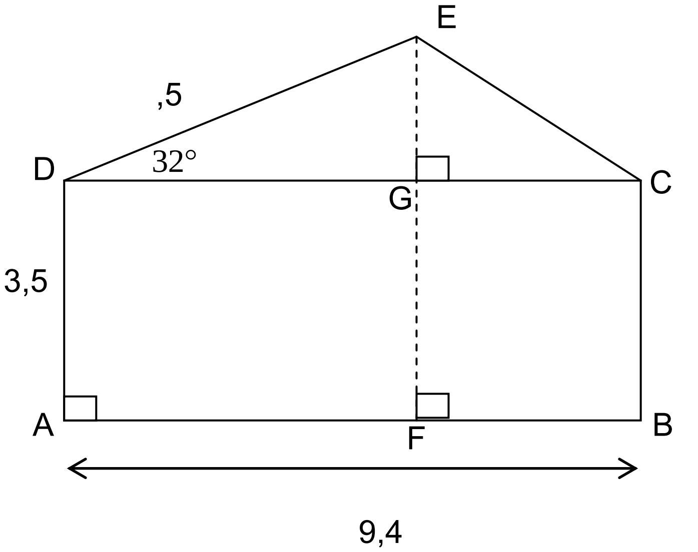
3.1 Determine AC (3)

3.2 Calculate ∠PAC (3)

3.3 A camera is positioned at point D, 40 metres directly below A. Calculate the distance from D to C. (4)
[10]

QUESTION 4 *(DOE March 2011)*

The sketch below shows one side of the elevation of a house. Some dimensions (in metres are indicated on the figure.)



Calculate, rounded off to ONE decimal place:

4.1 EC (3)

4.2 $D\hat{C}E$ (3)

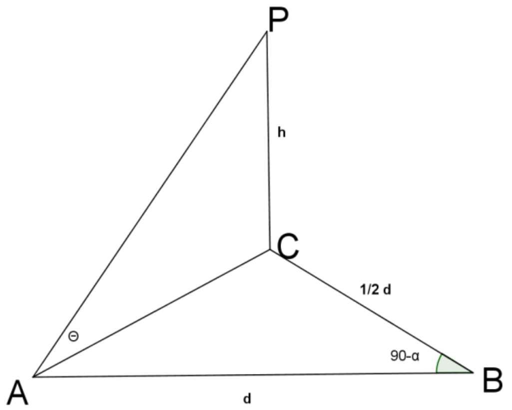
4.3 Area of ΔDEC (2)

4.4 The height EF (3)
[11]

SECTION B: HOMEWORK

QUESTION 1

In the diagram below the points ABC lie in the same horizontal plane. A hot air balloon is stationary at point P, which is directly above C. The angle of elevation of P from A is θ.
$\hat{ABC}$ =90 – α and the distance from B to C is half the distance of BA, where BA is d units.

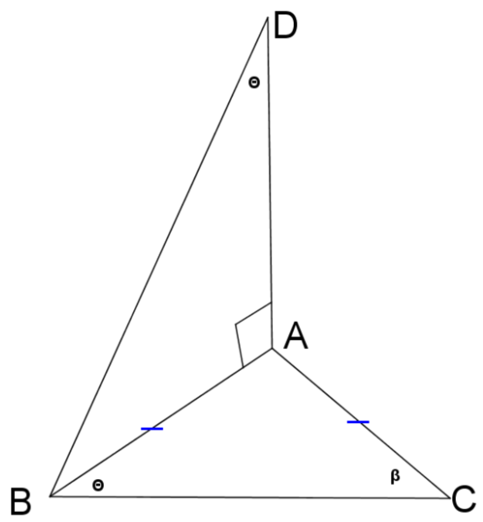


a) Show that the height of the balloon above C is $h = \frac{d\sqrt{(5-sin\alpha)}}{2}\ tan\theta$

b) Calculate h if d = 300, α =32 and θ = 63

QUESTION 2

In the figure A,B & C are three points in the same horizontal plane. AD represents a lamp pole that is perpendicular to the horizontal plane.



Given that ∠BDA = ∠ABC = θ and ∠BCA = β, BC =x

a) Write ∠BAC in terms of θ and β

b) Show that AB = $\frac{x\sin\beta}{\sin(\theta+\beta)}$

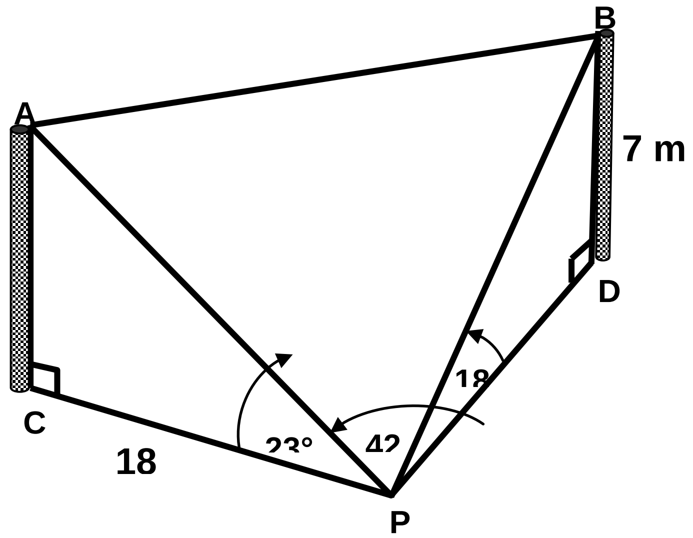
c) IF AB = AC show:

i) AB =$\frac{x}{2\cos\theta}$

ii) AD = $\frac{x}{2\sin\theta}$

QUESTION 3

Thandi is standing at point P on the horizontal ground and observes two poles, AC and BD, of different heights. P, C and D are in the same horizontal plane. From P the angles of inclination to the top of the poles A and B are 23° and 18° respectively. Thandi is 18 m from the base of pole AC. The height of pole BD is 7 m.



Calculate, correct to TWO decimal places:

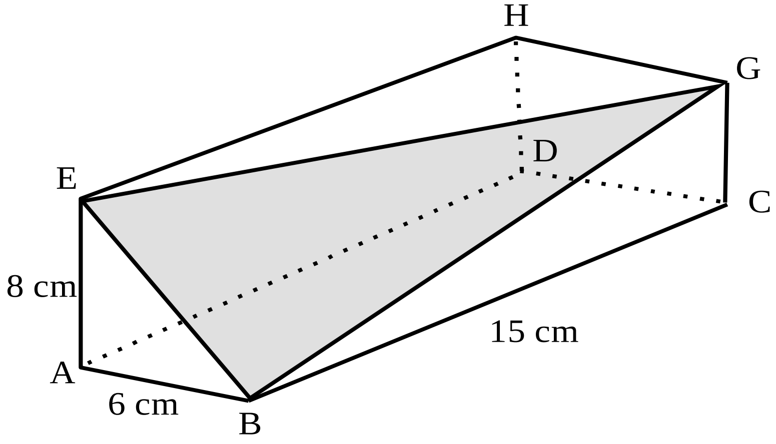
(a) The distance from Thandi to the top of pole BD.

(b) The distance from Thandi to the top of pole AC.

(c) The distance between the tops of the poles, that is the length of AB, if $A\hat{P}B = 42°$

QUESTION 4

A rectangular block of wood has a breadth of 6 metres, height of 8 metres and a length of 15 metres. A plane cut is made through the block as shown in the diagram revealing the triangular plane that has been formed. Calculate the size of $E\hat{B}G$.



SECTION B: SOLUTIONS TO SECTION A

QUESTION 1

(a) In ΔADB

ADB= 90-θ tanθ = $\frac{DA}{AB} = \frac{y}{AB}$ $\quad\quad \therefore AB = \frac{y}{tan\theta}$

In ΔABC

$$AC^2 = AB^2 + BC^2 - 2AB.BCcos120 = \left(\frac{y}{tan\theta}\right)^2 + y^2 - 2\left(\frac{y}{tan\theta}\right)(y)(-cos60)$$

$$=y^2\frac{}{\tan^2\theta} + y^2 + y^2\left(\frac{}{tan\theta}\right)$$

$$= y^2[\left(\frac{}{\tan^2\theta}\right) + 1 + \frac{}{tan\theta}]$$

$$\therefore AC = \sqrt{y^2\left[\left(\frac{}{\tan^2\theta}\right) + 1 + \frac{}{tan\theta}\right]} = y\sqrt{[\left(\frac{}{\tan^2\theta}\right) + 1 + \frac{}{tan\theta}]}$$ (7)

(b)

(i) AC = 46,4m (2)

(ii) In ΔADC: tan(ADC) = $\frac{AC}{AD} = \frac{46,48}{15} = 3,0985$ ....

ADC = 72.11˚ (3)

[12]

QUESTION 2

(a) In $\triangle$ADC:

$\hat{D} = 65°$ ($\angle s$ of Δ)

$$\frac{AC}{\sin 65°} = \frac{158}{\sin 25°}$$

$$\therefore AC.\sin 25° = 158.\sin 65°$$

$$\therefore AC = \frac{158.\sin 65°}{\sin 25°}$$

$$\therefore AC = 338,83\,m$$

(3)

(b) In $\triangle$ACB:

$$BC^2 = 338,83^2 + 1500^2 - 2(338,83)(1500)\cos 30°$$

$$\therefore BC^2 = 1\,484\,499,606$$

$$\therefore BC = 1218,4\,m$$

In $\triangle$DCB:

$$\tan\theta = \frac{DC}{BC}$$

$$\therefore \tan\theta = \frac{158}{1218,4}$$

$$\therefore \theta = 7,39°$$

(5)

(c)

$$\text{Area } \Delta ABC = \frac{1}{2}(338,83)(1500)\sin 30°$$

$$\therefore \text{Area } \Delta ABC = 127061,25m^2$$

(2)

(d)

$$AD^2 = (338,83)^2 + (158)^2$$

$$\therefore AD^2 = 139769,7689$$

$$\therefore AD = 373,86m$$

$$BD^2 = (1218,4)^2 + (158)^2$$

$$\therefore BD^2 = 1509462,56$$

$$\therefore BD = 1228,60m$$

$$(1500)^2 = (373,86)^2 + (1228,60)^2 - 2(373,86)(1228,60)\cos A\hat{D}B$$

$$\therefore 2(373,86)(1228,60)\cos A\hat{D}B = (373,86)^2 + (1228,60)^2 - (1500)^2$$

$$\therefore 918648,792\cos A\hat{D}B = -600770,7404$$

$$\therefore \cos A\hat{D}B = -0,6539721661$$

$$\therefore A\hat{D}B = 130,84°$$

(6)

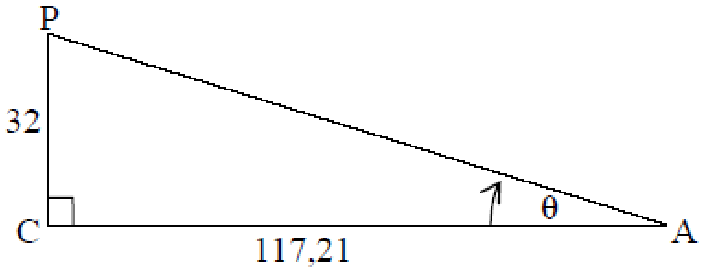
[16]

QUESTION 3

3.1 $\cos 64{,}75^\circ = \dfrac{50}{AC}$

$$AC = \frac{50}{\cos 64{,}75^\circ}$$
$$= 117{,}21\text{ m}$$ (3)

3.2 PC is given to be $\frac{1}{2}(64) = 32$ m



$$\tan P\hat{A}C = \frac{32}{117{,}21}$$
$$\theta = 15{,}27^\circ \quad (15{,}27042173...)$$

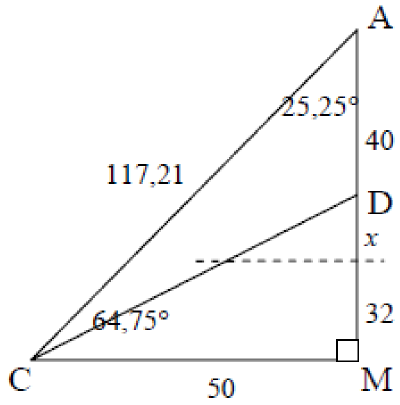
(3)

Note: If the candidate takes the unrounded answer for AC, then the answer is 15.27° (15.26987495...)

3.3 $$CD^2 = 117{,}21^2 + 40^2 - 2(117{,}21)(40)\cos 25{,}25$$
$$= 6857{,}289092$$
$$\therefore CD = 82{,}81 \quad m$$

Note:
If don't use the rounded off then CD = 82,81 m. Accept this answer.

OR



$$AM = AC\sin 64{,}75° \textbf{ OR } AM = CM\tan 64{,}75° \textbf{ OR } AM = AC\cos 25{,}25°$$
$$= 106{,}0111876 \qquad = 50\tan 64{,}75° \qquad = 117{,}21.\cos 25{,}25°$$
$$= 106{,}01 \qquad = 106{,}01 \qquad = 106{,}01$$
$$DM = 106{,}01 - 40$$
$$= 66{,}01$$
$$CD^2 = CM^2 + DM^2$$
$$= (50)^2 + (66{,}01)^2$$
$$= 6857{,}3201$$
$$CD = 82{,}81 \text{ metres}$$

(4)

[10]

QUESTION 4

4.1 $$EC^2 = DE^2 + DC^2 - 2DE.DC\cos\hat{C}$$
$$= (7{,}5)^2 + (9{,}4)^2 - 2.(7{,}5)(9{,}4)\cos 32°$$
$$= 25{,}03521844...$$
$$EC = 5{,}0 \text{ metres}$$

(3)

4.2 $$\frac{\sin DCE}{7{,}5} = \frac{\sin 32°}{5{,}0}$$
$$\sin D\hat{C}E = \frac{7{,}5.\sin 32°}{5{,}0}$$
$$= 0{,}7948788963$$
$$D\hat{C}E = 52{,}6°$$

(3)

4.3 Area of ΔDEC

$$= \frac{1}{2} DE.DC \sin \hat{D}$$

$$= \frac{1}{2}(7{,}5)(9{,}4)\sin 32^\circ$$

$$= 18{,}7m^2$$

OR

Area of ΔDEC

$$= \frac{1}{2} CE.DC \sin 52{,}6^\circ$$

$$= \frac{1}{2}(5{,}0)(9{,}4)\sin 52{,}6^\circ$$

$$= 18.7m^2$$

(2)

4.4

$$\sin 32^\circ = \frac{EG}{7{,}5}$$

$$EG = 7{,}5.\sin 32^\circ$$

$$= 4{,}0$$

$$EF = (4 + 3{,}5)$$

$$= 7{,}5 \text{ metres}$$

OR

$$EG = EC.\sin 52{,}6^\circ$$

$$= (5{,}0).\sin 52{,}6^\circ$$

$$= 4{,}0$$

$$EF = 4{,}0 + 3{,}5$$

$$= 7{,}5$$

(3)

[11]