SENIOR SECONDARY IMPROVEMENT PROGRAMME 2013

education
Department: Education
GAUTENG PROVINCE

GRADE 12

MATHEMATICS

TEACHER NOTES

The SSIP is supported by

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TEACHER NOTES

SESSION 20

TOPIC: REVISION OF ANALYTICAL GEOMETRY (GRADE 11)

Teacher Note: Analytical Geometry is an important topic that carries a lot of marks in the matric final exam. Make sure that learners know the basic formulae and then practise lots of examples involving applications of these formulae. The properties of quadrilaterals are extremely important in Analytical Geometry. Make sure learners can prove that a quadrilateral is a parallelogram, rectangle, square, rhombus or trapezium by knowing the properties of these quadrilaterals.

LESSON OVERVIEW

1.	Introduction session:	5 minutes
2.	Typical exam questions:	
	Question 1:	15 minutes
	Question 2:	15 minutes
	Question 3:	25 minutes
	Question 4:	5 minutes
	Question 5:	15 minutes
3.	Discussion of solutions:	10 minutes

SECTION A: TYPICAL EXAM QUESTIONS

QUESTION 1: 15 minutes

In the diagram, PQRS is a trapezium with vertices $P(5;2)$, $Q(1;-1)$, $R(9;-5)$ and S, and PS//QR. PT is the perpendicular height of PQRS and W is the midpoint of QR. Point S lies on the *x*-axis and $P\hat{R}Q = \theta$.

(a) Determine the equation of PW if W is the midpoint of QR. (2)
(b) Determine the equation of PS. (4)
(c) Determine the equation of PT. (3)
(d) Determine the coordinates of T. (5)
(e) Show that $QT = \frac{1}{3}TR$. (5)
(f) Calculate the size of θ rounded off to two decimal places. (5)
[24]

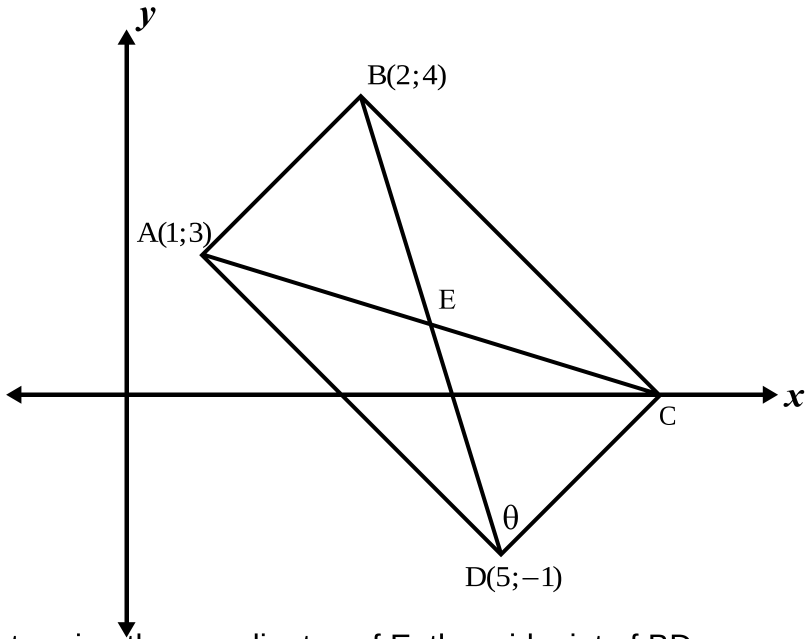
QUESTION 2: 15 minutes

Consider the following points on a Cartesian plane:
A(1;2), B(3;1), C(-3;*k*) and D(2;-3)
Determine the value(s) of k if:

(a) $(-1\,;3)$ is the midpoint of AC. (3)
(b) AB is parallel to CD. (3)
(c) AB $\perp$ CD. (3)
(d) A, B and C are collinear. (3)
(e) $CD = 5\sqrt{2}$ (5)
[17]

QUESTION 3: 25 minutes

ABCD is a quadrilateral with vertices $A(1;3)$, $B(2;4)$, C and $D(5;-1)$. The diagonals BD and AC bisect each other at point E



(a)	Determine the coordinates of E, the midpoint of BD.	(2)
(b)	Determine the coordinates of C.	(5)
(c)	Show that ABCD is a rectangle.	(10)
(d)	Determine the area of ABCD.	(6)
(e)	Calculate the size of the angle θ rounded off to the nearest degree.	(8)
		[31]

QUESTION 4: 5 minutes

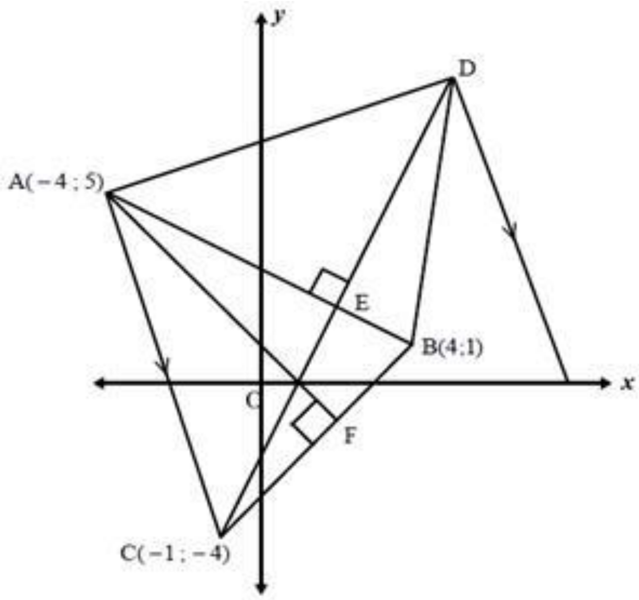
(a) Determine the numerical value of p if the straight line defined by the equation $px+3y=6$ has an angle of inclination of $135°$ with respect to the positive x-axis. (4)

(b) Calculate the value of k if the points $A(6;5)$, $B(3;2)$ and $C(2k;k+4)$ are collinear. (3)

[7]

QUESTION 5: 15 minutes

In the diagram on the following page, a(-4;5), C(-1;-4) and B94;1) are the vertices of a triangle in a Cartesian plane. CE⊥AB with E on AB. E is the midpoint of line CD. AF⊥BC with F on CB. The equation of AF is $x + y = 1$.

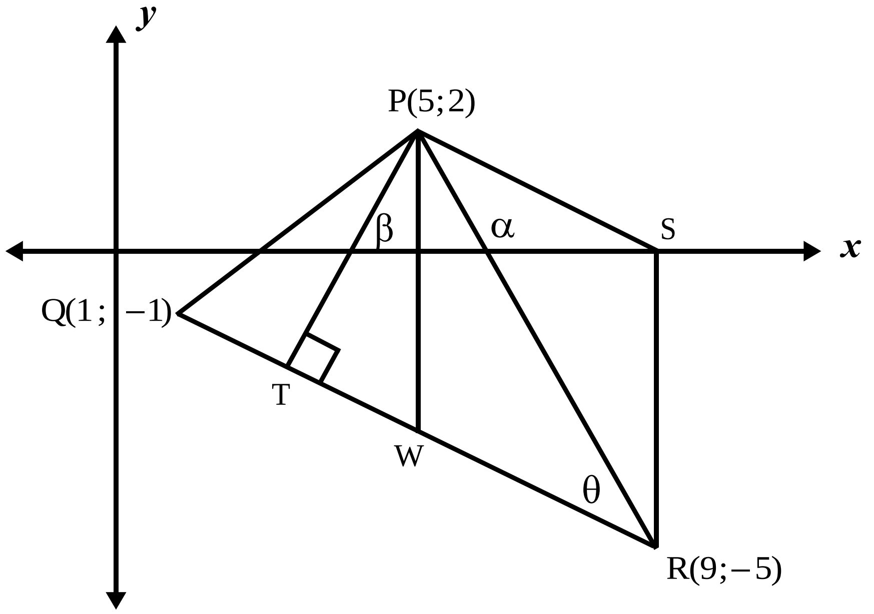


(a) Determine the equation of CD. (4)
(b) Determine the coordinates of E. (5)
(c) Determine the equation of the line through D and parallel to line AC. (6)
(d) Determine, showing all calculations, whether the *x*-intercept of line CD also lies on the line through AF with the equation $x + y = 1$. (3)

[18]

SECTION B: SOLUTIONS AND HINTS TO SECTION A

QUESTION 1



1(a)	$\text{W}\left(\frac{1+(9)}{2}\,;\,\frac{-1+(-5)}{2}\right)$ $=\text{W}(5\,;\,-3)$ The equation of PW is $x=5$	✓ midpoint ✓ $x=5$ (2)
1(b)	$m_{\text{QR}}=\frac{-5-(-1)}{9-1}=\frac{-4}{8}=-\frac{1}{2}$ $\therefore m_{\text{PS}}=-\frac{1}{2}$ (PS||QR) $y-2=-\frac{1}{2}(x-5)$ $\therefore y-2=-\frac{1}{2}x+\frac{5}{2}$ $\therefore y=-\frac{1}{2}x+\frac{9}{2}$	✓ m_{QR} ✓ m_{PS} ✓ correct substitution into formula for equation ✓ $y=-\frac{1}{2}x+\frac{9}{2}$ (4)
1(c)	$m_{\text{PT}}=2$ (PT ⊥ QR) $y-2=2(x-5)$ $\therefore y-2=2x-10$ $\therefore y=2x-8$	✓ m_{PT} ✓ correct substitution into formula for equation ✓ $y=2x-8$ (3)
1(d)	$m_{\text{QR}}=-\frac{1}{2}$ $y-(-1)=-\frac{1}{2}(x-1)$ $\therefore y+1=-\frac{1}{2}x+\frac{1}{2}$ $\therefore y=-\frac{1}{2}x-\frac{1}{2}$ $\therefore -\frac{1}{2}x-\frac{1}{2}=2x-8$ $\therefore -x-1=4x-16$ $\therefore -5x=-15$ $\therefore x=3$ $\therefore y=2(3)-8=-2$ $\therefore \text{T}(3;-2)$	✓ correct substitution into formula for equation ✓ $y=-\frac{1}{2}x-\frac{1}{2}$ ✓ $-\frac{1}{2}x-\frac{1}{2}=2x-8$ ✓ $x=3$ ✓ $\text{T}(3;-2)$ (5)

1(e)	$QT^2 = (1-3)^2 + (-1-(-2))^2$ $\therefore QT^2 = 4+1$ $\therefore QT^2 = 5$ $\therefore QT = \sqrt{5}$ $TR^2 = (3-9)^2 + (-2-(-5))^2$ $\therefore TR^2 = 36+9$ $\therefore TR^2 = 45$ $\therefore TR = \sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$ $\therefore \frac{1}{3}TR = \sqrt{5} = QT$ $\therefore QT = \frac{1}{3}TR$	✓ correct substitution to get QT ✓ answer for QT ✓ correct substitution to get TR ✓ answer for TR ✓ establishing that $QT = \frac{1}{3}TR$ (5)
1(f)	$\tan\alpha = m_{PR}$ $\therefore \tan\alpha = \frac{2-(-5)}{5-9}$ $\therefore \tan\alpha = -1$ $\therefore \alpha = 180° - 45°$ $\therefore \alpha = 135°$ $\tan\beta = m_{PT}$ $\therefore \tan\beta = 2$ $\therefore \beta = 63,43494882°$ Now $T\hat{P}R + \beta = \alpha$ $\therefore T\hat{P}R = \alpha - \beta$ $\therefore T\hat{P}R = 135° - 63,43494882°$ $\therefore T\hat{P}R = 71,56505118°$ $\theta + 90° + 71,56505118° = 180°$ $\therefore \theta = 18,43°$	✓ $\tan\alpha = -1$ ✓ $\alpha = 135°$ ✓ $\beta = 63,43494882°$ ✓ $T\hat{P}R = 71,56505118°$ ✓ $\theta = 18,43°$ (5)

[24]

QUESTION 2

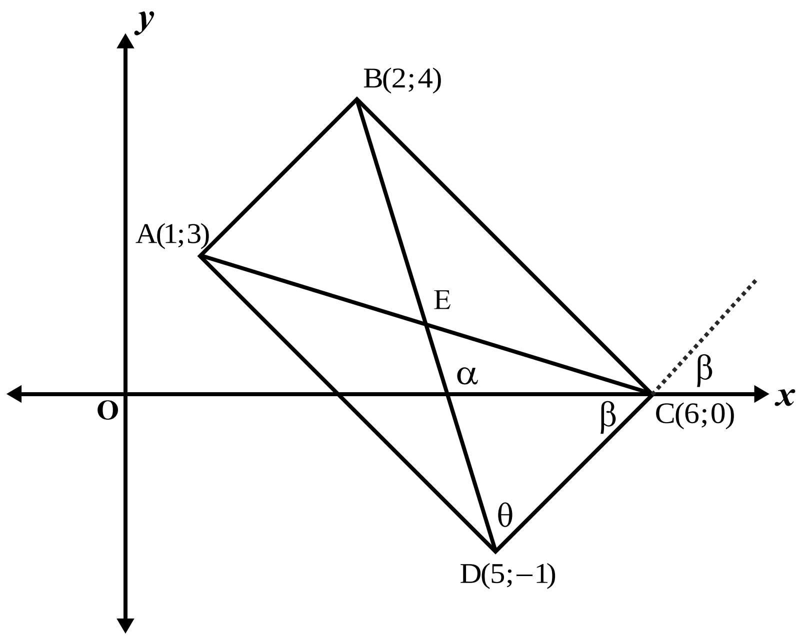
2(a)	$(-1;3)=\left(\frac{1+(-3)}{2};\frac{2+k}{2}\right)$ $(-1;3)=\left(-1;\frac{2+k}{2}\right)$ $\therefore 3=\frac{2+k}{2}$ $\therefore 6=2+k$ $\therefore k=4$	✓ $(-1;3)=\left(\frac{1+(-3)}{2};\frac{2+k}{2}\right)$ ✓ $\therefore 3=\frac{2+k}{2}$ ✓ $\therefore k=4$ (3)
2(b)	$m_{AB}=m_{CD}$ (lines //) $\therefore \frac{2-1}{1-3}=\frac{k-(-3)}{-3-2}$ $\therefore \frac{1}{-2}=\frac{k+3}{-5}$ $\therefore -5=-2(k+3)$ $\therefore -5=-2k-6$ $\therefore 2k=-1$ $\therefore k=-\frac{1}{2}$	✓✓ $\therefore \frac{2-1}{1-3}=\frac{k-(-3)}{-3-2}$ ✓ $\therefore k=-\frac{1}{2}$ (3)
2(c)	$m_{AB}\times m_{CD}=-1$ (lines $\perp$) $\therefore \frac{1}{-2}\times\frac{k+3}{-5}=-1$ $\therefore \frac{k+3}{10}=-1$ $\therefore k+3=-10$ $\therefore k=-13$	✓✓ $\therefore \frac{1}{-2}\times\frac{k+3}{-5}=-1$ ✓ $\therefore k=-13$ (3)
2(d)	$m_{AB}=m_{BC}$ (lines //) $\therefore \frac{1}{-2}=\frac{1-k}{3-(-3)}$ $\therefore \frac{1}{-2}=\frac{1-k}{6}$ $\therefore 6=-2(1-k)$ $\therefore 6=-2+2k$ $\therefore 8=2k$ $\therefore k=4$	✓✓ $\therefore \frac{1}{-2}=\frac{1-k}{3-(-3)}$ ✓ $\therefore k=4$ (3)
2(e)	$\text{CD}=5\sqrt{2}$ And $\text{CD}^2=(2-(-3))^2+(-3-k)^2$ $\therefore (5\sqrt{2})^2=5^2+9+6k+k^2$ $\therefore 50=25+9+6k+k^2$ $\therefore 0=k^2+6k-16$ $\therefore 0=(k+8)(k-2)$ $\therefore k=-8$ or $k=2$	✓ $\text{CD}^2=(2-(-3))^2+(-3-k)^2$ ✓ $\therefore (5\sqrt{2})^2=5^2+9+6k+k^2$ ✓ $\therefore 0=k^2+6k-16$ ✓ $\therefore 0=(k+8)(k-2)$ ✓ $\therefore k=-8$ or $k=2$ (5) **[17]**

QUESTION 3

3(a)	$E\left(\frac{2+5}{2};\frac{4+(-1)}{2}\right)$ $= E\left(\frac{7}{2};\frac{3}{2}\right)$	✓✓ $E\left(\frac{7}{2};\frac{3}{2}\right)$ (2)
3(b)	$E\left(\frac{7}{2};\frac{3}{2}\right)=\left(\frac{x_A+x_C}{2};\frac{y_A+y_C}{2}\right)$ $\therefore E\left(\frac{7}{2};\frac{3}{2}\right)=\left(\frac{1+x_C}{2};\frac{3+y_C}{2}\right)$ $\therefore \frac{7}{2}=\frac{1+x_C}{2}$ or $\frac{3}{2}=\frac{3+y_C}{2}$ $\therefore 7=1+x_C$ or $3=3+y_C$ $\therefore x_C=6$ or $y_C=0$ $\therefore C(6;0)$	✓ $\frac{7}{2}=\frac{1+x_C}{2}$ ✓ $\frac{3}{2}=\frac{3+y_C}{2}$ ✓ $x_C=6$ ✓ $y_C=0$ ✓ $C(6;0)$ (5)

3(c)	$m_{AB}=\frac{4-3}{2-1}=\frac{1}{1}=1$ $m_{CD}=\frac{0-(-1)}{6-5}=\frac{1}{1}=1$ $\therefore m_{AB}=m_{CD}$ $\therefore AB\parallel CD$ $m_{AD}=\frac{3-(-1)}{1-5}=\frac{4}{-4}=-1$ $m_{BC}=\frac{4-0}{2-6}=\frac{4}{-4}=-1$ $\therefore m_{AD}=m_{BC}$ $\therefore AD\parallel BC$ $\therefore$ ABCD is a parallelogram Now $m_{AB}\times m_{AD}=(1)\times(-1)=-1$ $\therefore AB\perp AD$ $\therefore \hat{A}=90°$ $\therefore$ ABCD is a rectangle (since one interior angle of parallelogram ABCD is 90°)	✓ m_{AB} ✓ m_{CD} ✓ $AB\parallel CD$ ✓ m_{AD} ✓ m_{BC} ✓ $AD\parallel BC$ ✓ $\therefore$ ABCD is a parallelogram ✓ $m_{AB}\times m_{AD}=-1$ ✓ $\hat{A}=90°$ ✓ ABCD is a rectangle (10)

3(d)	$AB^2 = (2-1)^2 + (4-3)^2$ $\therefore AB^2 = 1+1$ $\therefore AB^2 = 2$ $\therefore AB = \sqrt{2}$ $AD^2 = (5-1)^2 + (-1-3)^2$ $\therefore AD^2 = 16+16$ $\therefore AD = \sqrt{32} = \sqrt{16\times 2} = 4\sqrt{2}$ $\text{Area ABCD} = AD \times AB$ $\therefore \text{Area ABCD} = (4\sqrt{2})(\sqrt{2}) = 8 \text{ units}^2$	✓ $AB^2 = (2-1)^2 + (4-3)^2$ ✓ $AB = \sqrt{2}$ ✓ $AD^2 = (5-1)^2 + (-1-3)^2$ ✓ $AD = \sqrt{32}$ ✓ $\text{Area ABCD} = (4\sqrt{2})(\sqrt{2})$ ✓ $\text{Area ABCD} = 8 \text{ units}^2$ (6)



3(e)	$\tan\alpha = m_{BD}$ $\therefore \tan\alpha = \frac{4-(-1)}{2-5}$ $\therefore \tan\alpha = -\frac{5}{3}$ $\therefore \alpha = 180° - 59°$ $\therefore \alpha = 121°$	✓ $\tan\alpha = -\frac{5}{3}$ ✓ $\alpha = 121°$ ✓ $\tan\beta = 1$ ✓ $\beta = 45°$ ✓ $\theta = 18,43°$ ✓ $O\hat{C}D = \beta = 45°$ ✓ $121° = \theta + 45°$ ✓ $\theta = 76°$ (8)

	$\tan\beta = m_{DC}$ $\therefore \tan\beta = \frac{0-(-1)}{6-5} = \frac{1}{1} = 1$ $\therefore \tan\beta = 1$ $\therefore \beta = 45°$ $\therefore O\hat{C}D = \beta = 45°$ $\alpha = \theta + \beta$ $\therefore 121° = \theta + 45°$ $\therefore \theta = 76°$	

[31]

QUESTION 4

4(a)	$3y = -px + 6$ $\therefore y = -\frac{p}{3}x + 2$ $\therefore \tan 135° = -\frac{p}{3}$ $\therefore -1 = -\frac{p}{3}$ $\therefore p = 3$	✓ $y = -\frac{p}{3}x + 2$ ✓ $\tan 135° = -\frac{p}{3}$ ✓ $-1 = -\frac{p}{3}$ ✓ $p = 3$ (4)
4(b)	$m_{AB} = m_{BC}$ $\therefore \frac{2-5}{3-6} = \frac{k+4-2}{2k-3}$ $\therefore 1 = \frac{k+2}{2k-3}$ $\therefore 2k-3 = k+2$ $\therefore k = 5$	✓ $m_{AB} = m_{BC}$ ✓ working out gradients ✓ $k = 5$ (3)

[7]

QUESTION 5

5(a)	$m_{AB} = \frac{5-1}{-4-4} = -\frac{1}{2}$ $\therefore m_{CD} = 2$ $y — 4 = 2(x — 1)$ $\therefore y + 4 = 2x + 2$ $\therefore y = 2x - 2$	✓ m_{AB} ✓ m_{CD} ✓ correct substitution into formula for equation of line ✓ $y = 2x - 2$ (4)
5(b)	Equation of AB $y - 1 = -\frac{1}{2}(x - 4)$ $\therefore y - 1 = -\frac{1}{2}x + 2$ $\therefore y = -\frac{1}{2}x + 3$ Equation of CD $y = 2x - 2$ $\therefore -\frac{1}{2}x + 3 = 2x - 2$ $\therefore -x + 6 = 4x - 4$ $\therefore -5x = -10$ $\therefore x = 2$ $\therefore y = 2$ $\therefore E(2; 2)$	✓ $y - 1 = -\frac{1}{2}(x - 4)$ ✓ $y = -\frac{1}{2}x + 3$ ✓ $-\frac{1}{2}x + 3 = 2x - 2$ ✓ $x = 2$ ✓ E(2;2) (5)
5(c)	C(-1;-4) E(2;2) D$(x_D; y_D)$ $2 = \frac{-1+x_D}{2}$ and $2 = \frac{-4+y_D}{2}$ $4 = -1 + x_D$ $\therefore x_D = 5$ $4 = -4 + y_D$ $\therefore y_D = 8$ $D(5; 8)$ Now $m_{AC} = \frac{5-(-4)}{-4-(-1\_} = \frac{9}{-3} = -3$ Equation of line required: $y - 8 = -3(x - 5)$ $\therefore y - 8 = -3x + 15$ $\therefore y = -3x + 23$	✓ $x_D = 5$ ✓ $y_D = 8$ ✓ m_{AB} ✓ m_{CD} ✓ correct substitution into formula for equation of line ✓ $y = 2x - 2$ (6)
5(d)	x-intercept: $0 = 2x - 2$ $\therefore 2 = 2x$ $\therefore x = 1$ $(1; 0)$ Substitute (1;0) into $x + y = 1$ $LHS = x + y = 1 + 0 = 1 = RHS$ $\therefore (1; 0)$ lies on the line AF	✓ $x = 1$ ✓ Substitute (1;0) ✓ LHS=RHS (3)

[18]

SECTION C: HOMEWORK

QUESTION 1 6 marks

A(-1;4), B(1;-2) and C(9;2) are vertices of triangle ABC. Find

(a) the equation of BC (3)

(b) the equation of the line perpendicular to BC, passing through A. (3)

[6]

QUESTION 2 6 marks

The points A(-5;4) and B(7;-2) are given. Find the acute angle between the line AB and the line given by $5x + y + 5 = 0$ (6)

[6]

QUESTION 3 8 marks

PQ is the line segment that joins the points P(-1;3) and Q(5;-7). Find

a) the coordinates of the midpoint of PQ (2)

b) the length of PQ (2)

c) the equation of the perpendicular bisector of PQ (4)

[8]

SECTION D: SOLUTIONS TO HOMEWORK

1(a)	$m_{BC} = \frac{2+2}{9-1}$ $m_{BC} = \frac{1}{2}$ $y + 2 = \frac{1}{2}(x-1)$ $y = \frac{1}{2}x - 2\frac{1}{2}$	✓ $m_{BC} = \frac{1}{2}$ ✓ $y + 2 = \frac{1}{2}(x-1)$ ✓ $y = \frac{1}{2}x - 2\frac{1}{2}$ (3)
1(b)	$m = -2$ $y - 4 = -2(x+1)$ $y = -2x + 2$	✓ $m = -2$ ✓ $y - 4 = -2(x+1)$ ✓ $y = -2x + 2$ (3) **[6]**
2	$m_{AB} = \frac{4+2}{-5+7}$ $m_{AB} = 3$ $\tan\alpha = 3$ $\therefore \alpha = 71{,}57°$ $5x + y + 5 = 0$ $y = -5x - 5$ $m = -5$ $\tan\beta = -5$ $\beta = 180° - 78{,}69°$ $\beta = 101{,}31°$ $\theta = 101{,}31° - 71{,}57°$ $\theta = 29{,}74°$	✓$m_{AB} = 3$ ✓ $\alpha = 71{,}57°$ ✓$m = -5$ ✓$\beta = 180° - 78{,}69°$ ✓$\theta = 101{,}31° - 71{,}57°$ ✓$\theta = 29{,}74°$ **[6]**
3(a)	$\left(\frac{-1+5}{2}; \frac{3-7}{2}\right)$ $(2; -2)$	✓$\left(\frac{-1+5}{2}; \frac{3-7}{2}\right)$ ✓$(2; -2)$ (2)
3(b)	$d_{PQ} = \sqrt{(-1-5)^2 + (3+7)^2}$ $d_{PQ} = 11{,}66$	✓$d_{PQ} = \sqrt{(-1-5)^2 + (3+7)^2}$ ✓ $d_{PQ} = 11{,}66$ (2)
3(c)	$m_{PQ} = \frac{-7-3}{5+1}$ $m_{PQ} = \frac{-5}{3}$ $\therefore new\ gradient = \frac{3}{5}$ $y + 2 = \frac{3}{5}(x-2)$ $y = \frac{3}{5}x - \frac{16}{5}$	✓$m_{PQ} = \frac{-5}{3}$ ✓$\therefore new\ gradient = \frac{3}{5}$ ✓ $y + 2 = \frac{3}{5}(x-2)$ ✓ $y = \frac{3}{5}x - \frac{16}{5}$ (4) **[8]**