SENIOR SECONDARY INTERVENTION PROGRAMME 2013

education
Department: Education
GAUTENG PROVINCE

GRADE 12

MATHEMATICS

LEARNER HOMEWORK SOLUTIONS

The SSIP is supported by

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LEARNER HOMEWORK SOLUTIONS

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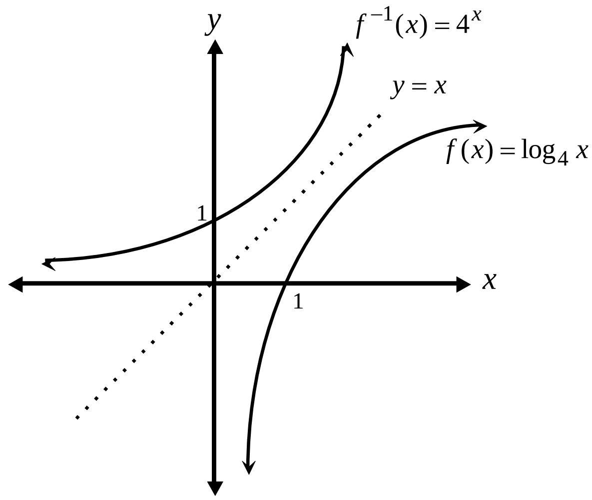
SOLUTIONS TO HOMEWORK: SESSION 1
TOPIC 1: LOGARITHMS

1	$0,125=\left(\frac{1}{2}\right)^{\frac{n}{5700}}$ $\therefore 0,125=(0,5)^{\frac{n}{5700}}$ $\therefore \log_{0,5} 0,125=\frac{n}{5700}$ $\therefore 3=\frac{n}{5700}$ $\therefore n=17100$ years	✓ $0,125$ ✓ $0,125=(0,5)^{\frac{n}{5700}}$ ✓ $\log_{0,5} 0,125=\frac{n}{5700}$ ✓ $n=17100$ years **[4]**
2(a)	$0,5x=x(1-0,08)^n$ $\therefore 0,5=0,92^n$ $\therefore \log_{0,92} 0,5=n$ $\therefore n=8,312950414$ 8 years and 4 months	✓ correct substitution into formula ✓ use of logs ✓ answer (3)
2(b)	$A=P\left(1+\frac{i}{12}\right)^{12n}$ $100\ 000=50\ 000\left(1+\frac{0,18}{12}\right)^{12n}$ $\therefore 2=(1,015)^{12n}$ $\therefore \log_{1,015} 2=12n$ $\therefore 12n=46,55552563$ $\therefore n=3,879627136$ 3 years 11 months Divide i by 12 and multiply n by 12	✓ $12n$ ✓ $\frac{0,18}{12}$ ✓ $2=(1,015)^{12n}$ ✓ $\log_{1,015} 2=12n$ ✓ $n=3,879627136$ ✓ 3 years 11 months (6) **[9]**
3(a)	$\therefore 2=\log_a 16$ $\therefore a^2=16$ $\therefore a=4$	✓ $2=\log_a 16$ ✓ $a^2=16$ ✓ $a=4$ (3)

3(b)	$y = \log_4 x$ $\therefore x = \log_4 y$ $\therefore 4^x = y$ $\therefore f^{-1}(x) = 4^x$	✓ $x = \log_4 y$ ✓ $f^{-1}(x) = 4^x$ (2)
3(c)		✓ Graph of $f(x)$ ✓ y-intercept of f ✓ Graph of $f^{-1}(x)$ ✓ x-intercept of f^{-1} (4) **[9]**

SOLUTIONS TO HOMEWORK: SESSION 1
TOPIC 2: FACTORISATION OF THIRD DEGREE POLYNOMIALS

1(a)	$x^3-6x^2-x-6=0$ $\therefore (x-1)(x^2-5x-6)=0$ $\therefore (x-1)(x-6)(x+1)=0$ $\therefore x=\pm1$ or $x=6$	✓ $(x-1)$ ✓ (x^2-5x-6) ✓ $(x-6)(x+1)$ ✓ $x=\pm1$ or $x=6$ (4)
1(b)	$x^3-2x^2+16=0$ $\therefore (x+2)(x^2-4x+8)=0$ $\therefore x=-2$ or $x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(8)}}{2(1)}$ $x=\frac{4\pm\sqrt{-16}}{2}$ non-real	✓ $(x+2)$ ✓ (x^2-4x+8) ✓ $x=-2$ ✓ $x=\frac{4\pm\sqrt{-16}}{2}$ ✓ $x=-2$ is the only real solution (5)
1(c)	$2x^3-5x^2-4x+3=0$ $\therefore (x+1)(2x^2-7x+3)=0$ $\therefore (x+1)(2x-1)(x-3)=0$ $\therefore x=-1$ or $x=\frac{1}{2}$ or $x=3$	✓ $(x+1)$ ✓ $(2x^2-7x+3)$ ✓ $(2x-1)(x-3)$ ✓ $x=-1$ or $x=\frac{1}{2}$ or $x=3$ (4)

1(d)	$-x^3+4x^2-2x-4=0$ $\therefore x^3-4x^2+2x+4=0$ $\therefore (x-2)(x^2-2x-2)=0$ $\therefore x=2$ or $x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-2)}}{2(1)}$ $x=\frac{2\pm\sqrt{12}}{2}$ $x=2,73$ or $x=-0,73$	✓ $(x-2)$ ✓ (x^2-2x-2) ✓ $x=2$ ✓ $x=\frac{2\pm\sqrt{12}}{2}$ ✓ $x=2,73$ ✓ $x=-0,73$ (6)
1(e)	$x^3-5x^2-3x+9=0$ $\therefore (x+1)(x^2-6x+9)=0$ $\therefore (x+1)(x-3)^2=0$ $\therefore x=-1$ or $x=3$	✓ $(x+1)$ ✓ (x^2-6x+9) ✓ $(x-3)^2$ ✓ $x=-1$ or $x=3$ (4)

[23]

2(a)	$0=x^3-8x^2+19x-12$ $\therefore (x-1)(x^2-7x+12)=0$ $\therefore (x-1)(x-4)(x-3)=0$ $\therefore x=1$ or $x=4$ or $x=3$ $(1;0)$ $(4;0)$ $(3;0)$	✓ $0=x^3-8x^2+19x-12$ ✓ $(x-1)$ ✓ $(x^2-7x+12)$ ✓ $(x-4)(x-3)$ ✓ $x=1$ or $x=4$ or $x=3$ ✓ $(1;0)$ $(4;0)$ $(3;0)$ (6)
2(b)	$0=x^3-x^2-x-2$ $\therefore (x-2)(x^2+x+1)=0$ $\therefore x=2$ or $x=\frac{-1\pm\sqrt{(1)^2-4(1)(1)}}{2(1)}$ $x=\frac{-1\pm\sqrt{-3}}{2}$ non-real	✓ $(x-2)$ ✓ (x^2+x+1) ✓ $x=2$ ✓ $x=\frac{-1\pm\sqrt{-3}}{2}$ ✓ $x=2$ is the only x-intercept (5)

[11]

SOLUTIONS TO HOMEWORK: SESSION 2
TOPIC 1: SEQUENCES & SERIES

1(a)	$T_{19} = a + 18d = 11$ $T_{31} = a + 30d = 5$ $\therefore 12d = -6$ $\therefore d = -\frac{1}{2}$ $\therefore a + 18(\frac{-1}{2}) = 11$ $\therefore a = 20$ $\therefore 20;\ 19\frac{1}{2};\ 19...$	✓ $T_{19} = a + 18d = 11$ ✓ $T_{31} = a + 30d = 5$ ✓ $d = -\frac{1}{2}$ ✓ $a = 20$ ✓ sequence (5)
1(b)	$20 + (n-1)\left(-\frac{1}{2}\right) = -29$ $\therefore (n-1)\left(-\frac{1}{2}\right) = -49$ $\therefore n - 1 = 98$ $\therefore n = 99$ $\therefore T_{99} = -29$	✓ Substitution into formula ✓ equating to -29 ✓ $n = 99$ (3)
2(a)	$\frac{1}{181} + \frac{2}{181} + \frac{3}{181} + \frac{4}{181} + ... + \frac{180}{181}$ $a = \frac{1}{181} \quad d = \frac{1}{181} \quad n = 180$ $S_{180} = \frac{180}{2}\left[2\left(\frac{1}{181}\right) + (179)\frac{1}{181}\right] = 90[1] = 90$	✓ correct *a* and *d* ✓ correct *n* ✓ S_n formula ✓ correct answer (4)
2(b)	$\left(\frac{1}{2}\right) + \left(\frac{1}{3} + \frac{2}{3}\right) + \left(\frac{1}{4} + \frac{2}{4} + \frac{3}{4}\right) + ... + \left(\frac{1}{181} + \frac{2}{181} + ... + \frac{180}{181}\right)$ $= \frac{1}{2} + 1 + 1\frac{1}{2} + 2 + ... + 90 \quad [a = \frac{1}{2} \quad d = \frac{1}{2} \quad T_n = 90]$ $\therefore \frac{1}{2} + (n-1)\frac{1}{2} = 90$ $\therefore 1 + n - 1 = 180$ $\therefore n = 180$ $\therefore S_{180} = \frac{180}{2}\left[\frac{1}{2} + 90\right] = 90\left[90\frac{1}{2}\right] = 8145$	✓ simplifying fractions to get series ✓ $\frac{1}{2} + (n-1)\frac{1}{2} = 90$ ✓ $n = 180$ ✓ substitution into S_n formula to get 8145 (4)

3(a)	$(2p-3)-(1-p)=(p+5)-(2p-3)$ $2p-3-1+p=p+5-2p+3$ $3p-4=-p+8$ $4p=12$ $p=3$	✓ $T_2-T_1=T_3-T_2$ ✓ $2p-3-1+p=p+5-2p+3$ ✓ $p=3$ (3)
3(b)(1)	$T_1=1-(3)=-2$	✓ answer (1)
3(b)(2)	$d=T_2-T_1=(2p-3)-(1-p)$ $\therefore d=(2(3)-3)-(1-3)$ $\therefore d=3-(-2)$ $\therefore d=5$	✓ $(2p-3)-(1-p)$ ✓ $p=3$ (2)
3(c)	The sequence is $-2;3;8;13;18;23;28;33;38;.......$ After the first term -2, all the other terms end in either a 3 or an 8. Perfect squares never end in a 3 or an 8.	✓✓ answer (2)
4	$ar^5=\sqrt{3}$ $ar^7=\sqrt{27}$ $\therefore \frac{ar^7}{ar^5}=\frac{\sqrt{27}}{\sqrt{3}}$ $\therefore r^2=\sqrt{\frac{27}{3}}$ $\therefore r^2=\sqrt{9}$ $\therefore r^2=3$ $\therefore r=\sqrt{3}$ (terms are positive) $\therefore a(\sqrt{3})^5=\sqrt{3}$ $\therefore a=\frac{\sqrt{3}}{(\sqrt{3})^5}$ $\therefore a=\frac{1}{(\sqrt{3})^4}$ $\therefore a=\frac{1}{(3^{\frac{1}{2}})^4}$ $\therefore a=\frac{1}{9}$	✓ $ar^5=\sqrt{3}$ ✓ $ar^7=\sqrt{27}$ ✓ dividing ✓ $r^2=3$ ✓ $r=\sqrt{3}$ ✓ correct working with surds ✓ $a=\frac{1}{9}$ (7)

5(a)	$\sum_{r=1}^{n}(6r-1)=[6(1)-1]+[6(2)-1]+[6(3)-1]+.....+[6(n)-1]=456$ $=5+11+17+....+(6n-1)=456$ $S_n=\frac{n}{2}(2a+(n-1)d)$ $\therefore 456=\frac{n}{2}(2a+(n-1)d)$ $\therefore 456=\frac{n}{2}(2(5)+(n-1)(6))$ $\therefore 456=\frac{n}{2}(10+6n-6)$ $\therefore 456=\frac{n}{2}(4+6n)$ $\therefore 456=2n+3n^2$ $\therefore 0=3n^2+2n-456$ $\therefore (3n+38)(n-12)=0$ $\therefore 3n=-38 \text{ or } n=12$ $\therefore n=-\frac{38}{3} \text{ or } n=12$ $\therefore n=12$	✓ expanding ✓ correct formula ✓ $456=\frac{n}{2}(2a+(n-1)d)$ ✓ $0=3n^2+2n-456$ ✓ $(3n+38)(n-12)=0$ ✓ $n=-\frac{38}{3}$ or $n=12$ ✓ $\therefore n=12$ (7)
5(b)	$\sum_{k=3}^{n}[(2k-1)n]=5n+7n+9n+...+[(2n-1)n]$ $\therefore a=5n,\ d=2n$ and number of terms $=n-2$ $\therefore S_{n-2}=\frac{n-2}{2}[2a+(n-2-1)d]$ $\therefore S_{n-2}=\frac{n-2}{2}[2(5n)+(n-3)(2n)]$ $\therefore S_{n-2}=\frac{n-2}{2}[10n+2n^2-6n]$ $\therefore S_{n-2}=\frac{n-2}{2}[2n^2+4n]$ $\therefore S_{n-2}=2n(n-2)+n^2(n-2)$ $\therefore S_{n-2}=2n^2-4n+n^3-2n^2=n^3-4n$	✓ expanding ✓ $a=5n,\ d=2n$ ✓ number of terms $=n-2$ ✓ correct formula ✓ substitution ✓ answer (6)

6(a)	$\sum_{n=1}^{\infty} 2(\frac{1}{2}x)^n$ $= 2(\frac{1}{2}x)^1 + 2(\frac{1}{2}x)^2 + 2(\frac{1}{2}x)^3 + 2(\frac{1}{2}x)^4 + \ldots\ldots\ldots\ldots$ $= x + \frac{1}{2}x^2 + \frac{1}{4}x^3 + \frac{1}{8}x^4 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots$ The series converges for: $-1 < \frac{1}{2}x < 1$ $\therefore -2 < x < 2$	✓ $r = \frac{1}{2}x$ ✓ $-1 < \frac{1}{2}x < 1$ ✓ $-2 < x < 2$ (3)
6(b)	$a = \frac{1}{2} \quad r = \frac{1}{2}\left(\frac{1}{2}\right) = \frac{1}{4}$ $\therefore S_{\infty} = \frac{\frac{1}{2}}{1-\frac{1}{4}} = \frac{2}{3}$	✓ a and r ✓ S_{∞} formula ✓ $\frac{2}{3}$ (3)

Question 7:

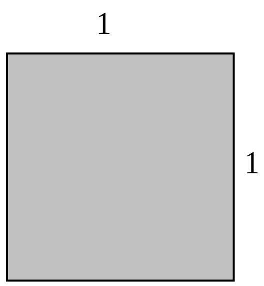


DIAGRAM 1

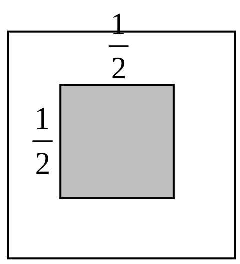


DIAGRAM 2

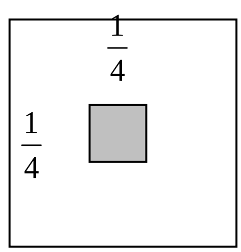


DIAGRAM 3

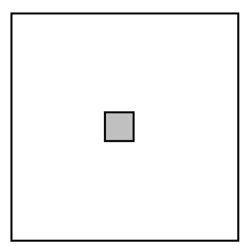


DIAGRAM 4

7(a)	Area of unshaded square $=$ Area of large square $-$ Area of small shaded square $= (1)(1) - \left(\frac{1}{4}\right)\left(\frac{1}{4}\right)$ $= 1 - \frac{1}{16} = \frac{15}{16}$	✓ $(1)(1) - \left(\frac{1}{4}\right)\left(\frac{1}{4}\right)$ ✓ $\frac{15}{16}$ (2)

7(b)	Sum of the unshaded areas of the first seven squares: $=(1-1)+\left(1-\frac{1}{4}\right)+\left(1-\frac{1}{4^2}\right)+\cdots+\left(1-\frac{1}{4^6}\right)$ $=7-\left(1+\frac{1}{4}+\frac{1}{4^2}+\cdots+\frac{1}{4^6}\right)$ $=7-\left(\frac{1\left(1-\left(\frac{1}{4}\right)^7\right)}{1-\frac{1}{4}}\right)$ $=7-1,333251953...$ $=5,67$	✓ ✓ Getting the pattern for the unshaded areas ✓ correct formula ✓ substitution ✓ answer (5)
8	$\therefore S_\infty=\frac{1,5}{1-\left(\frac{2}{3}\right)}$ $\therefore S_\infty=4,5\,\text{m}$ Thus the greatest height is 4,5 m	✓ correct formula ✓ substitution ✓ answer (3)

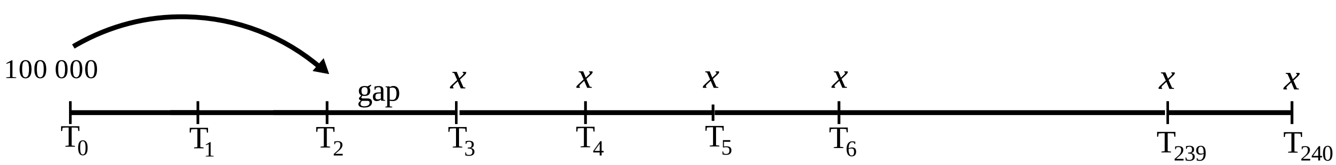
SOLUTIONS TO HOMEWORK: SESSION 3

FINANCIAL MATHEMATICS

QUESTION 1

(a)	$2\ 500\ 000 = \dfrac{x\left[(1{,}0075)^{361} - 1)\right]}{0{,}0075}$ $\therefore \dfrac{2\ 500\ 000 \times 0{,}0075}{\left[(1{,}0075)^{361} - 1)\right]} = x$ $\therefore x = \text{R}1354{,}67$	✓ correct formula ✓ $n = 361$ ✓ $\dfrac{0{,}09}{12} = 0{,}0075$ ✓ $\text{F} = 2\ 500\ 000$ ✓ answer (5)
(b)	$2\ 500\ 000 = \dfrac{x\left[1 - \left(1 + \dfrac{0{,}07}{12}\right)^{-240}\right]}{\left(\dfrac{0{,}07}{12}\right)}$ $\therefore \dfrac{2\ 500\ 000 \times \left(\dfrac{0{,}07}{12}\right)}{\left[1 - \left(1 + \dfrac{0{,}07}{12}\right)^{-240}\right]} = x$ $\therefore x = \text{R}19\ 382{,}47$	✓ correct formula ✓ $n = 240$ ✓ $\dfrac{0{,}07}{12}$ ✓ $\text{P} = 2\ 500\ 000$ ✓ answer (5)

QUESTION 2



(a)	$\therefore 100\ 000(1{,}015)^2 = \dfrac{x\left[1 - (1{,}015)^{-238}\right]}{0{,}015}$ $\therefore 100\ 000(1{,}015)^2 \times 0{,}015 = x\left[1 - (1{,}015)^{-238}\right]$ $\therefore \dfrac{100\ 000(1{,}015)^2 \times 0{,}015}{\left[1 - (1{,}015)^{-238}\right]} = x$ $\therefore x = \text{R}1591{,}35$	✓ correct formula ✓ $n = 238$ ✓ $\dfrac{0{,}18}{12} = 0{,}015$ ✓ $100\ 000(1{,}015)^2$ ✓ answer (5)

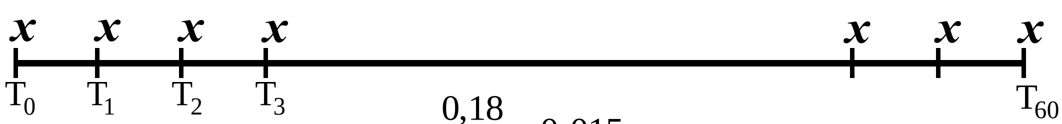
SOLUTIONS TO HOMEWORK: SESSION 4

FINANCIAL MATHS

QUESTION 1

1(a)	$A = 200\ 000(1-0,2)^5$ $\therefore A = R65\ 536$	✓ correct formula ✓ answer (2)
1(b)	$A = 200\ 000(1+0,16)^5$ $\therefore A = R420\ 068,33$	✓ correct formula ✓ answer (2)
1(c)	Sinking fund $= 420\ 068,33 - 65\ 536$ Sinking fund $= 354\ 532,33$	✓ answer (1)

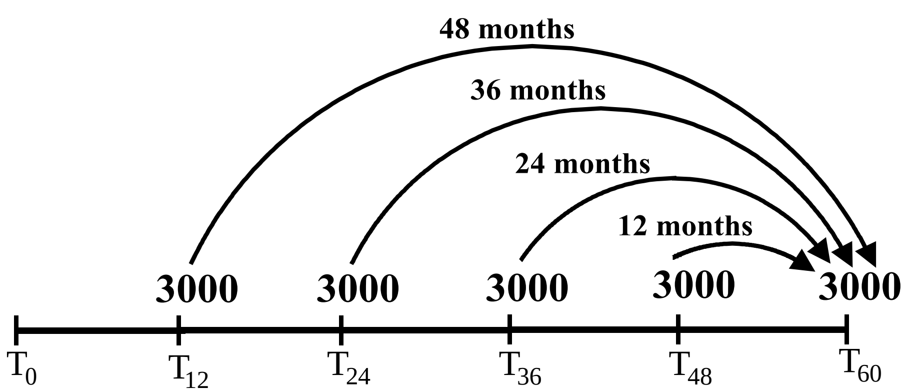
1(d) Draw a time line



$\frac{0,18}{12} = 0,015$

1(d)	$354\ 532,33 = \frac{x\left[(1,015)^{61}-1\right]}{0,015}$ $\frac{354\ 532,33 \times 0,015}{\left[(1,015)^{61}-1\right]} = x$ $\therefore x = R3593,55$	✓ correct formula ✓ $F = 354\ 532,33$ ✓ $n = 61$ ✓ $\frac{0,18}{12} = 0,015$ ✓ $x = R3593,55$ (4)

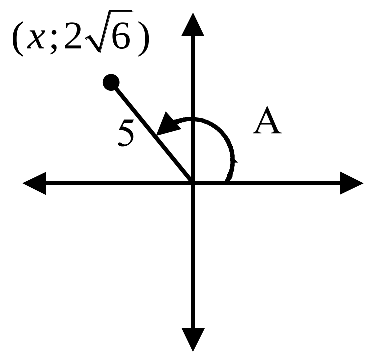
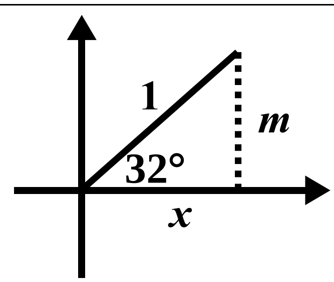
1(e)(1) Draw a time line



1(e)(1)	Future value of the withdrawals: $3000\left(1+\frac{0,18}{12}\right)^{48}+3000\left(1+\frac{0,18}{12}\right)^{36}$ $+3000\left(1+\frac{0,18}{12}\right)^{24}+3000\left(1+\frac{0,18}{12}\right)^{12}+3000$ $=\text{R22 133,22}$ The reduced value of the sinking fund will be: $\text{R354 532,33}-\text{R22 133,22}=\text{R332 399,11}$	✓ services ✓ R22 133,22 ✓ reduced value (3)
1(e)(2)	If we add R22 133,22 to the original sinking fund amount of R354 532,33, then it will be possible not only to receive the sinking fund amount of R354 532,33 at the end of the five year period, but also to be able to make the service withdrawals at the end of each year for the five year period. $354\ 532{,}33+22\ 133{,}22=\frac{x\left[(1,015)^{61}-1\right]}{0,015}$ $\therefore 376\ 665{,}55=\frac{x\left[(1,015)^{61}-1\right]}{0,015}$ $\therefore \frac{376\ 665{,}55\times 0,015}{\left[(1,015)^{61}-1\right]}=x$ $\therefore x=\text{R3817,90}$	✓ correct formula ✓ $F=376\ 665{,}55$ ✓ $n=61$ ✓ $\frac{0,18}{12}=0,015$ ✓ $x=\text{R3817,90}$ (5)
2(a)	$100\ 000=\frac{2000\left[(1,08)^{n+1}-1\right]}{0,08}$ $\therefore \frac{100\ 000\times 0,08}{2000}=(1,08)^{n+1}-1$ $\therefore \frac{100\ 000\times 0,08}{2000}+1=(1,08)^{n+1}$ $\therefore 5=(1,08)^{n+1}$ $\therefore \log_{1,08}5=n+1$ $\therefore n=19,91237188$ $\therefore n\approx 20$ half-years $\therefore n\approx 10$ years (since there are 20 half years in a ten-year period)	✓ correct formula ✓ $\frac{0,16}{2}=0,08$ ✓ time period $=n+1$ ✓ $5=(1,08)^{n+1}$ ✓ $\log_{1,08}5=n+1$ ✓ $n=19,91237188$ ✓ $n\approx 10$ years (7)

2(b)	$400\ 000=\frac{17000\left[1-(1,04)^{-n}\right]}{0,04}$ $\therefore \frac{400\ 000\times 0,04}{17000}=1-(1,04)^{-n}$ $\therefore (1,04)^{-n}=1-\frac{400\ 000\times 0,04}{17000}$ $\therefore (1,04)^{-n}=0,05882352941$ $\therefore 0,05882352941=(1,04)^{-n}$ $\therefore \log_{1,04} 0,05882352941=-n$ $\therefore n=72,23768046$ 18 years 3 months	✓ correct formula ✓ $\frac{0,16}{4}=0,04$ ✓ $0,0588...=(1,04)^{-n}$ ✓ $\log_{0,0625} 1,015=-n$ ✓ $n=72,23768046$ ✓ answer (6)

SOLUTIONS TO HOMEWORK: SESSION 5
TRIGONOMETRY (REVISION)

1(a)	$\sin A = \frac{2\sqrt{6}}{5}$ $x^2 = r^2 - y^2$ $x^2 = 5^2 - \left(2\sqrt{6}\right)^2$ $x^2 = 1$ $x = \pm 1$ $\therefore x = -1$  $\therefore 5\tan A.\cos A = 5\left(\frac{2\sqrt{6}}{-1}\right).\left(\frac{-1}{5}\right) = 2\sqrt{6}$	✓ $\sin A = \frac{2\sqrt{6}}{5}$ ✓ correct quadrant ✓ $x = -1$ ✓✓correct substitution: $5\left(\frac{2\sqrt{6}}{-1}\right).\left(\frac{-1}{5}\right)$ ✓ correct answer (6)
1(b)(1)	$\sin 32° = \frac{m}{1}$ $x^2 + m^2 = 1$ $\therefore x = \sqrt{1-m^2}$ $\sin 328° = \sin 32° = m$ 	✓ correct quadrant ✓ $x = \sqrt{1-m^2}$ ✓ $\sin 328° = \sin 32°$ ✓ answer (4)
1(b)(2)		✓ $\sin 32°$ ✓ answer (2)
1(b)(3)	$\tan 212°$ $= \tan 32°$ $= \frac{m}{\sqrt{1-m^2}}$	✓ $\tan 32°$ ✓ $\frac{m}{\sqrt{1-m^2}}$ (2)
2(a)	$\frac{\cos 210°.\tan^2 315°}{\sin 300°.\cos 120°}$ $= \frac{(-\cos 30°)(-\tan 45°)^2}{(-\sin 60°)(-\cos 60°)}$ $= \frac{\left(-\frac{\sqrt{3}}{2}\right)(-1)^2}{\left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2}\right)}$ $= -2$	✓ $-\cos 30°$ ✓ $(-\tan 45°)^2$ ✓ $-\sin 60°$ ✓ $-\cos 60°$ ✓ evaluating special angles ✓ answer (6)

2(b)	$\frac{\sin^2(180°+\theta).\sin(360°-\theta)}{\cos^2(90°-\theta).\cos(90°+\theta)}$ $=\frac{(\sin^2\theta)(-\sin\theta)}{(\sin^2\theta)(-\sin\theta)}$ $=1$	Numerator: ✓ $\sin^2\theta$ ✓ $-\sin\theta$ Denominator: ✓ $\sin^2\theta$ ✓ $-\sin\theta$ ✓ 1 (5)
2(c)	$\cos^2(360°-x)-\sin(180°-x)\cos(90°+x)-\cos^2(180+x)$ $=\cos^2 x-(\sin x)(-\sin x)-\cos^2 x$ $=\sin^2 x$	✓✓✓✓reductions ✓ $\sin^2 x$ (5)
3(a)	$\frac{\tan(180°+x)\cos(360°-x)}{\sin(180°-x)\cos(90°+x)+\cos(540°+x)\cos(-x)}$ $=\frac{(\tan x)(\cos x)}{(\sin x)(-\sin x)-(\cos x)(\cos x)}$ $=\frac{\frac{\sin x}{\cos x}\cos x}{-\sin^2 x-\cos^2 x}$ $=\frac{\sin x}{-(\sin^2 x+\cos^2 x)}$ $=-\sin x$	Numerator: ✓✓ $(\tan x)(\cos x)$ ✓ $\frac{\sin x}{\cos x}$ Denominator: ✓✓✓✓ $(\sin x)(-\sin x)-(\cos x)(\cos x)$ ✓ $-(\sin^2 x+\cos^2 x)$ ✓ $-\sin x$ (9)
3(b)	$\frac{\cos(-64°)\tan(-244°)\sin^2 334°}{\cos 566°}$ $=\frac{(\cos 64°)(-\tan 244°)(-\sin 26°)^2}{\cos 206°}$ $=\frac{(\cos 64°)(-\tan 64°)\sin^2 26°}{-\cos 26°}$ $=\frac{(\cos 64°)\left(\frac{-\sin 64°}{\cos 64°}\right)\sin^2 26°}{-\sin 64°}$ $=\sin^2 26°$ $=1-\cos^2 26°$ $=1-p^2$	✓ $\cos 64°$ ✓ $-\tan 64°$ ✓ $\sin^2 26°$ ✓ $-\cos 26°$ ✓ $\frac{-\sin 64°}{\cos 64°}$ ✓ $-\sin 64°$ ✓ $1-\cos^2 26°$ ✓ $1-p^2$ (8)

4	$\cos^2 x\left[\frac{1}{\sin x-1}+\frac{1}{\sin x+1}\right]$ $=\cos^2 x\left[\frac{(1+\sin x)+(1-\sin x)}{(1+\sin x)(1-\sin x)}\right]$ $=\cos^2 x\left[\frac{2}{1-\sin^2 x}\right]$ $=\cos^2 x\left[\frac{2}{\cos^2 x}\right]$ $=2$	✓ $(1+\sin x)+(1-\sin x)$ ✓ $(1+\sin x)(1-\sin x)$ ✓ $\frac{2}{1-\sin^2 x}$ ✓ $\frac{2}{\cos^2 x}$ ✓ 2 (5)

SOLUTIONS TO HOMEWORK: SESSION 6
TRIGONOMETRY

1(a)	$\dfrac{\sin 34°\cos 10° - \cos 34°\sin 10°}{\sin 12°\cos 12°}$ $= \dfrac{\sin(34° - 10°)}{\sin 12°\cos 12°}$ $= \dfrac{\sin 24°}{\sin 12°\cos 12°}$ $= \dfrac{2\sin 12°\cos 12°}{\sin 12°\cos 12°} = 2$	✓ $\sin 24°$ ✓ $2\sin 12°\cos 12°$ ✓ 2 (3)
1(b)	$\sin(-285°)$ $= -\sin 285°$ $= -\sin(360° - 75°)$ $= -(-\sin 75°)$ $= \sin 75°$ $= \sin(45° + 30°)$ $= \sin 45°\cos 30° + \cos 45°\sin 30°$ $= \left(\dfrac{\sqrt{2}}{2}\right)\left(\dfrac{\sqrt{3}}{2}\right) + \left(\dfrac{\sqrt{2}}{2}\right)\left(\dfrac{1}{2}\right) = \dfrac{\sqrt{6}+\sqrt{2}}{4}$	✓ $-\sin 285°$ ✓ $\sin 75°$ ✓ $\sin 45°\cos 30° + \cos 45°\sin 30°$ ✓ $\left(\dfrac{\sqrt{2}}{2}\right)\left(\dfrac{\sqrt{3}}{2}\right) + \left(\dfrac{\sqrt{2}}{2}\right)\left(\dfrac{1}{2}\right)$ ✓ $\dfrac{\sqrt{6}+\sqrt{2}}{4}$ (5)
1(c)	$\dfrac{\cos^2 15° - \sin 15°\cos 75°}{\cos^2 15° + \sin 15°\cos 15°\tan 15°}$ $= \dfrac{\cos^2 15° - \sin 15°\cos(90° - 15°)}{\cos^2 15° + \sin 15°\cos 15°\left(\dfrac{\sin 15°}{\cos 15°}\right)}$ $= \dfrac{\cos^2 15° - \sin 15°.\sin 15°}{\cos^2 15° + \sin^2 15°}$ $= \dfrac{\cos^2 15° - \sin^2 15°}{1}$ $= \cos^2 15° - \sin^2 15°$ $= \cos 2(15°)$ $= \cos 30° = \dfrac{\sqrt{3}}{2}$	✓ $\cos^2 15° - \sin 15°.\sin 15°$ ✓ $\dfrac{\sin 15°}{\cos 15°}$ ✓ $\cos^2 15° + \sin^2 15°$ ✓ 1 ✓ $\cos 30°$ ✓ $\dfrac{\sqrt{3}}{2}$ (6)

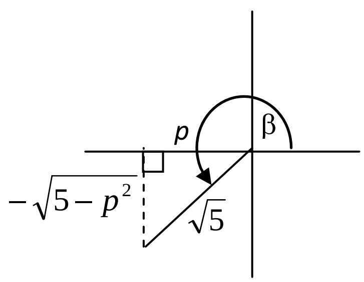
[14]

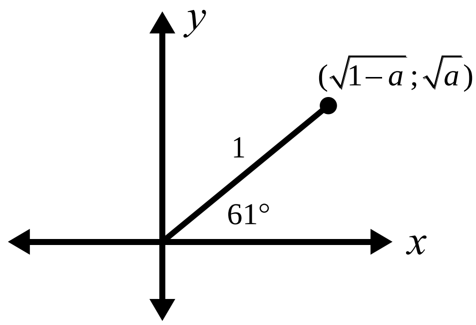
2(a)	$\sin(45°+\theta).\sin(45°-\theta)$ $=[\sin 45°\cos\theta+\cos 45°\sin\theta][\sin 45°\cos\theta-\cos 45°\sin\theta]$ $=\left[\frac{\sqrt{2}}{2}\cos\theta+\frac{\sqrt{2}}{2}\sin\theta\right]\left[\frac{\sqrt{2}}{2}\cos\theta-\frac{\sqrt{2}}{2}\sin\theta\right]$ $=\left[\frac{\sqrt{2}}{2}(\cos\theta+\sin\theta)\right]\left[\frac{\sqrt{2}}{2}(\cos\theta-\sin\theta)\right]$ $=\frac{2}{4}(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)$ $=\frac{1}{2}(\cos^2\theta-\sin^2\theta)$ $=\frac{1}{2}\cos 2\theta$	✓ expansion of $\sin(45°+\theta)$ ✓ expansion of $\sin(45°-\theta)$ ✓ $\sin 45°=\cos 45°=\frac{\sqrt{2}}{2}$ ✓ $(\cos^2\theta-\sin^2\theta)$ ✓ $\frac{1}{2}\cos 2\theta$ (5)
2(b)	$\sin 75°.\sin 15°$ $=\sin(45°+30°).\sin(45-30°)$ $=\frac{1}{2}\cos 2(30°)$ $=\frac{1}{2}\cos 60°=\frac{1}{2}\left(\frac{1}{2}\right)=\frac{1}{4}$	✓ $45°+30°$; $45°-30°$ ✓ $\frac{1}{2}\cos 60°$ ✓ $\frac{1}{4}$ (3) **[8]**
3	$\sin 4\theta$ $=\sin 2(2\theta)$ $=2\sin 2\theta.\cos 2\theta$ $=2(2\sin\theta.\cos\theta)(1-2\sin^2\theta)$ $=4\sin\theta.\cos\theta-8\sin^3\theta.\cos\theta$	✓ $2\sin 2\theta.\cos 2\theta$ ✓ $2\sin\theta.\cos\theta$ ✓ $1-2\sin^2\theta$ ✓ $4\sin\theta.\cos\theta-8\sin^3\theta.\cos\theta$ **[4]**

SOLUTIONS TO HOMEWORK: SESSION 7
TOPIC 2: TRIGONOMETRY

1(a)	$(\tan x - 1)(\sin 2x - 2\cos^2 x)$ $= \left(\frac{\sin x}{\cos x} - 1\right)\left(2\sin x.\cos x - 2\cos^2 x\right)$ $= \left(\frac{\sin x - \cos x}{\cos x}\right) 2\cos x(\sin x - \cos x)$ $= 2(\sin x - \cos x)^2$ $= 2(\sin^2 x - 2\sin x.\cos x + \cos^2 x)$ $= 2(1 - 2\sin x.\cos x)$	✓ $\frac{\sin x}{\cos x}$ ✓ $2\sin x \cos x$ ✓ $\frac{\sin x - \cos x}{\cos x}$ ✓ $2\cos x(\sin x - \cos x)$ ✓ $2(\sin^2 x - 2\sin x.\cos x + \cos^2 x)$ ✓ $2(1 - 2\sin x.\cos x)$ (6)
1(b)	$\frac{\cos 2x}{\cos x - \sin x}$ $= \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x}$ $= \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x - \sin x}$ $= \cos x + \sin x$	✓ $\cos^2 x - \sin^2 x$ ✓ $(\cos x - \sin x)(\cos x + \sin x)$ ✓ $\cos x + \sin x$ (3) [9]
2(a)	$\sin(45° - \alpha)$ $= \sin 45°.\cos\alpha - \cos 45°.\sin\alpha$ $= \frac{\sqrt{2}}{2}.\cos\alpha - \frac{\sqrt{2}}{2}.\sin\alpha$ $= \frac{\sqrt{2}}{2}(\cos\alpha - \sin\alpha)$ $= \frac{\sqrt{2}(\cos\alpha - \sin\alpha)}{2}$	✓ $\sin 45°.\cos\alpha - \cos 45°.\sin\alpha$ ✓ $\frac{\sqrt{2}}{2}.\cos\alpha - \frac{\sqrt{2}}{2}.\sin\alpha$ ✓ $\frac{\sqrt{2}}{2}(\cos\alpha - \sin\alpha)$ (3)

2(b)	$\sin 2\alpha + 2\sin^2(45° - \alpha)$ $= 2\sin\alpha.\cos\alpha + 2\left[\sin(45° - \alpha)\right]^2$ $= 2\sin\alpha.\cos\alpha + 2\left[\frac{\sqrt{2}(\cos\alpha - \sin\alpha)}{2}\right]^2$ $= 2\sin\alpha.\cos\alpha + 2\left(\frac{2(\cos\alpha - \sin\alpha)^2}{4}\right)$ $= 2\sin\alpha.\cos\alpha + (\cos\alpha - \sin\alpha)^2$ $= 2\sin\alpha.\cos\alpha + \cos^2\alpha - 2\sin\alpha\cos\alpha + \sin^2\alpha$ $= \cos^2\alpha + \sin^2\alpha = 1$	✓ $2\sin\alpha.\cos\alpha$ ✓ $2\left[\frac{\sqrt{2}(\cos\alpha - \sin\alpha)}{2}\right]^2$ ✓ $(\cos\alpha - \sin\alpha)^2$ ✓ $\cos^2\alpha - 2\sin\alpha\cos\alpha + \sin^2\alpha$ ✓ $\cos^2\alpha + \sin^2\alpha$ ✓ 1 (6) **[9]**

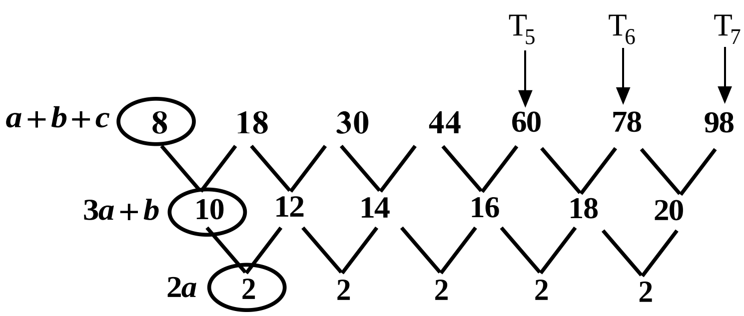
3(a)	$\cos\beta = \frac{p}{\sqrt{5}}$ $x = p$ $r = \sqrt{5}$ $p^2 + y^2 = (\sqrt{5})^2$ $\therefore y^2 = 5 - p^2$ $\therefore y = -\sqrt{5 - p^2}$ $\therefore \tan\beta = \frac{-\sqrt{5 - p^2}}{p}$  	✓ diagram ✓ Pythagoras ✓ $y = -\sqrt{5 - p^2}$ ✓ $\tan\beta = \frac{-\sqrt{5 - p^2}}{p}$ (4)
3(b)	$\cos 2\beta = 2\cos^2\beta - 1$ $= 2\left(\frac{p}{\sqrt{5}}\right)^2 - 1$ $= \frac{2p^2}{5} - 1$	✓ $\cos 2\beta = 2\cos^2\beta - 1$ ✓ $2\left(\frac{p}{\sqrt{5}}\right)^2 - 1$ ✓ $\frac{2p^2}{5} - 1$ (3) **[7]**

4	$\sin 61° = \frac{\sqrt{a}}{1}$ $x^2 + (\sqrt{a})^2 = (1)^2$ $\therefore x^2 = 1 - a$ $\therefore x = \sqrt{1-a}$  $\cos 73° \cos 15° + \sin 73° \sin 15°$ $= \cos(73° - 15°)$ $= \cos 58°$ $= 2\cos^2 29° - 1$ $= 2\sin^2 61° - 1$ $= 2(\sqrt{a})^2 - 1$ $= 2a - 1$	✓ diagram ✓ $x = \sqrt{1-a}$ ✓ $\cos 58°$ ✓ $2\cos^2 29° - 1$ ✓ $2\sin^2 61° - 1$ ✓ $2a - 1$ **[6]**

SOLUTIONS TO SESSION 8 CONSOLDATION

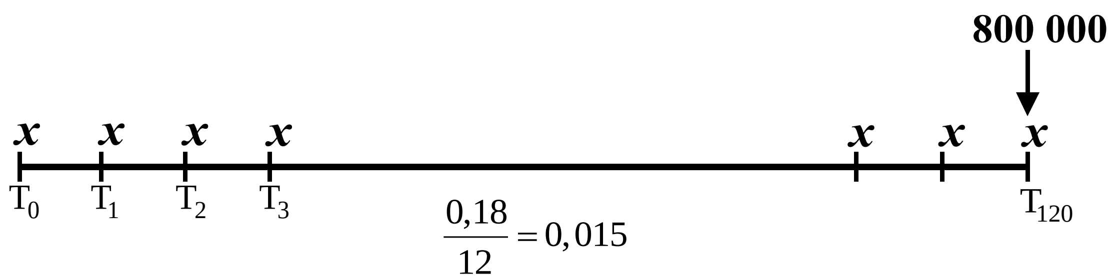
TOPIC 1: NUMBER PATTERNS, SEQUENCES AND SERIES

QUESTION 1

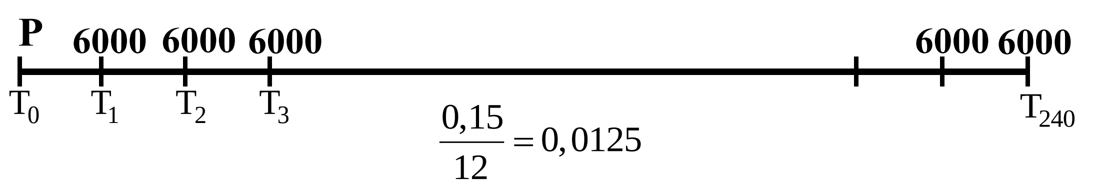
1.1.1	$\frac{1}{16}$; 13	✓ answers (1)
1.1.2	$S_{50} =$ 25 terms of 1st sequence + 25 terms of 2nd sequence $S_{50} = \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \text{... to 25 terms}\right) + \left(4 + 7 + 10 + 13 + \text{... to 25 terms}\right)$ $S_{50} = \frac{\frac{1}{2}\left(\left(\frac{1}{2}\right)^{25} - 1\right)}{\frac{1}{2} - 1} + \frac{25}{2}\left[2(4) + 24(3)\right]$ $S_{50} = 0,999999... + 1000$ $S_{50} = 1001,00$	✓ separating the sequences ✓✓ sum of geometric series ✓✓ sum of arithmetic series ✓✓ final answer (7)
1.2.1	60 ; 78	✓✓ answers (2)
1.2.2	 $\therefore 2a = 2$ $3a + b = 10$ $a + b + c = 8$ $\therefore a = 1$ $\therefore 3(1) + b = 10$ $\therefore 1 + 7 + c = 8$ $\therefore b = 7$ $\therefore c = 0$ $\therefore T_n = n^2 + 7n$	✓ first difference row ✓ second difference ✓ $a = 1$ ✓ $b = 7$ ✓ $c = 0$ ✓ $T_n = n^2 + 7n$ (6)

1.2.3	$330 = n^2 + 7n$ $\therefore -n^2 - 7n + 330 = 0$ $\therefore n^2 + 7n - 330 = 0$ $\therefore n = \dfrac{-7 \pm \sqrt{(7)^2 - 4(1)(-330)}}{2}$ $\therefore n = \dfrac{-7 \pm 37}{2}$ $\therefore n = 15$ or $n = -22$ (not valid) Alternatively, factorise: $(n-15)(n+22) = 0$	✓ $330 = n^2 + 7n$ ✓ $n^2 + 7n - 330 = 0$ ✓✓ Substitution into formula or factorising ✓ $n = 15$ or $n = -22$ ✓ $n \neq -22$ (6) **[22]**

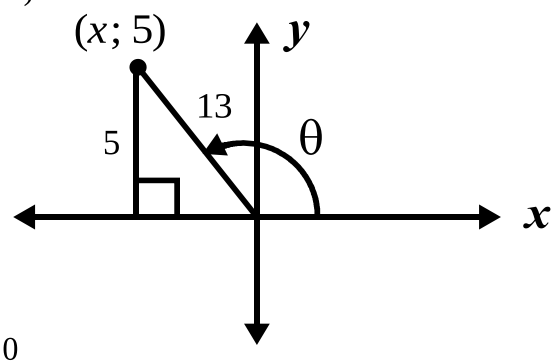
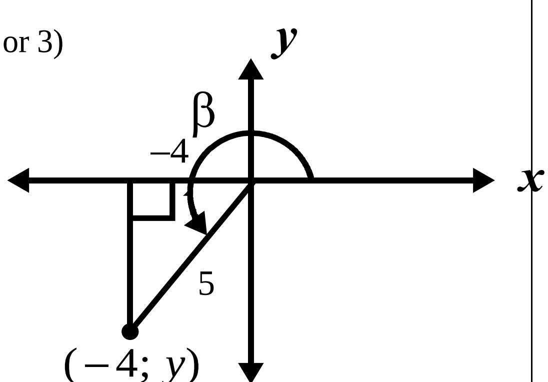
TOPIC 2: FINANCIAL MATHS AND TRIGONOMETRY



1.1.1	$800\ 000 = \dfrac{x\left[\left(1+\dfrac{0,18}{12}\right)^{121} - 1\right]}{\dfrac{0,18}{12}}$ $\therefore x = \text{R}2372,07$	✓ formula ✓ $n = 121$ ✓ $\dfrac{0,18}{12}$ ✓ $\text{F} = 800\ 000$ ✓ answer (5)
1.1.2	$\text{A} = 800\ 000\left(1+\dfrac{0,18}{2}\right)^8$ $\therefore \text{A} = \text{R}1\ 594\ 050,11$	✓ formula ✓ $n = 8$ ✓ $\dfrac{0,18}{2}$ ✓ answer (4)



1.2	$P = \dfrac{6000\left[1-\left(1+\frac{0,15}{12}\right)^{-240}\right]}{\frac{0,15}{12}}$ $\therefore x = R455\ 653,67$	✓ formula ✓ $n = 240$ ✓ $\frac{0,15}{12}$ ✓ answer (4)

2.1.1	$13\sin\theta - 5 = 0$ $\therefore \sin\theta = \frac{5}{13}$ (positive in Quad 1 or 2) $\theta \in [90°\,;360°]$ (Quad 2, 3 or 4) Common quad is 2 $x^2 + (5)^2 = (13)^2$ $\therefore x^2 = 144$ $\therefore x = -12$  $5\cos\beta + 4 = 0$ $\therefore \cos\beta = \frac{-4}{5}$ (negative in Quad 2 or 3) $\tan\beta > 0$ (Quad 1 or 3) Common quad is 3 $(-4)^2 + y^2 = (5)^2$ $\therefore y^2 = 9$ $\therefore y = -3$ 	✓ $\sin\theta = \frac{5}{13}$ ✓ diagram for θ ✓ $x = -12$ ✓ $\cos\beta = \frac{-4}{5}$ ✓ diagram for β ✓ $y = -3$ ✓ $\sin\theta\cos\beta - \cos\theta\sin\beta$ ✓ correct substitution ✓ answer (9)

	$\sin(\theta-\beta)$ $=\sin\theta\cos\beta-\cos\theta\sin\beta$ $=\left(\frac{5}{13}\right)\left(\frac{-4}{5}\right)-\left(\frac{-12}{13}\right)\left(\frac{-3}{5}\right)$ $=\frac{-20}{65}-\frac{36}{65}$ $=-\frac{56}{65}$	
2.1.2	$\tan(90°+\theta)$ $=\frac{\sin(90°+\theta)}{\cos(90°+\theta)}$ $=\frac{\cos\theta}{-\sin\theta}$ $=\frac{\frac{-12}{13}}{-\frac{5}{13}}$ $=\frac{12}{5}$	✓ $\frac{\sin(90°+\theta)}{\cos(90°+\theta)}$ ✓ $\cos\theta$ ✓ $-\sin\theta$ ✓ correct substitution ✓ answer (5)

2.2.1	$\cos(50°+x)\cos(20°+x)+\sin(50°+x)\sin(20°+x)$ $=\cos[(50°+x)-(20°+x)]$ $=\cos 30°$ $=\frac{\sqrt{3}}{2}$	✓ $\cos[(50°+x)-(20°+x)]$ ✓ $\cos 30°$ ✓ $\frac{\sqrt{3}}{2}$ (3)
2.2.2	$\cos(-140°)\cos 740°-\sin 140°\sin(-20°)$ $=\cos 140°\cos 20°-(\sin 40°)(-\sin 20°)$ $=(-\cos 40°)(\cos 20°)+\sin 40°\sin 20°$ $=-\cos 40°\cos 20°+\sin 40°\sin 20°$ $=-(\cos 40°\cos 20°-\sin 40°\sin 20°)$ $=-\cos 60°$ $=-\frac{1}{2}$	✓ $-\cos 40°$ ✓ $\cos 20°$ ✓ $\sin 40°$ ✓ $-\sin 20°$ ✓ $-\cos 60°$ ✓ $-\frac{1}{2}$ (6) **[23]**

SOLUTIONS TO HOMEWORK: SESSION 9
TOPIC 1: NUMBER PATTERNS, SEQUENCES AND SERIES

QUESTION 1

1.1	$\sum_{r=3}^{17} 10 = (\text{number of terms}) \times 10$ $= (17-3+1)\times 10$ $= 15\times 10$ $= 150$	✓ 150 (1)
1.2.1	$a = 20$ $a+(n-1)d = 136$ $\therefore 20+(30-1)d = 136$ $\therefore 20+29d = 136$ $\therefore 29d = 116$ $\therefore d = 4$ There are 24 seats in the second row.	✓ $a = 20$ ✓ $20+29d = 136$ ✓ $d = 4$ ✓ 24 seats in second row ✓ $S_{30} = 2340$ (4)
1.2.2	$S_{30} = \frac{30}{2}[20+136] = 2340$ seats	✓ $S_{30} = 2340$ (1)
1.3.1	$r = \frac{x}{12}$ $\frac{12}{1-\frac{x}{12}} = 24$ $\therefore 12 = 24\left(1-\frac{x}{12}\right)$ $\therefore 12 = 24-2x$ $\therefore 2x = 12$ $\therefore x = 6$	✓ $r = \frac{x}{12}$ ✓ $\frac{12}{1-\frac{x}{12}} = 24$ ✓ $x = 6$ (3)
1.3.2	$12\left(\frac{1}{2}\right)^{n-1} = \frac{3}{128}$ $\therefore \left(\frac{1}{2}\right)^{n-1} = \frac{1}{512}$ $\therefore \left(\frac{1}{2}\right)^{n-1} = \left(\frac{1}{2}\right)^{9}$ $\therefore n-1 = 9$ $\therefore n = 10$	✓ $12\left(\frac{1}{2}\right)^{n-1} = \frac{3}{128}$ ✓ $\left(\frac{1}{2}\right)^{n-1} = \frac{1}{512}$ ✓ $n = 10$ (3) **[12]**

QUESTION 2

2.1	Area of triangle 1: $\frac{1}{2}(2cm)(2cm) = (1)(2)cm^2$ Area of triangle 2: $\frac{1}{2}(4cm)(3cm) = (2)(3)cm^2$ Area of triangle 3: $\frac{1}{2}(6cm)(4cm) = (3)(4)cm^2$ Area of triangle 4: $\frac{1}{2}(8cm)(5cm) = (4)(5)cm^2$ The areas form the following pattern: $(1)(2);(2)(3);(3)(4);(4)(5);..........$ Area of triangle *n*: $(n)(n+1)cm^2$ Area of triangle 100: $(100)(100+1)cm^2 = 10100cm^2$	✓ determining areas ✓ establishing pattern ✓ obtaining general term ✓ area of 100th triangle (4)
2.2	$n(n+1) = 240$ $\therefore n^2 + n - 240 = 0$ $\therefore (n+16)(n-15) = 0$ $\therefore n = -16$ or $n = 15$ But $n \neq -16$ $\therefore n = 15$ The 15th triangle will have an area of $240\,\text{cm}^2$	✓ correct *a* and *d* ✓ correct *n* ✓ S_n formula ✓ correct answer (4) **[8]**

SOLUTIONS TO HOMEWORK: SESSION 11
TOPIC 2: FINANCIAL MATHEMATICS

QUESTION 1

1.1	$3000000 = \dfrac{x\left[\left(1+\frac{0,12}{12}\right)^{241}-1\right]}{\frac{0,12}{12}}$ $\therefore x = \text{R}2999,56$	✓ formula ✓ correct substitution ✓ answer (3)
1.2.1	$250\ 000\left(1+\frac{0,185}{12}\right)^{3} = \dfrac{x\left[1-(1+\frac{0,185}{12})^{-57}\right]}{\frac{0,185}{12}}$ $\therefore x = \text{R }6\ 934,45$	✓ correct formula ✓ $250\ 000\left(1+\frac{0,185}{12}\right)^{3}$ ✓ $n = 57$ ✓ $\frac{0,185}{12}$ ✓ answer (5)
1.2.2	$\text{B} = 250\ 000\left(1+\frac{0,185}{12}\right)^{28} - \dfrac{6\ 934,45\left[\left(1+\frac{0,185}{12}\right)^{25}-1\right]}{\frac{0,185}{12}}$ $\therefore \text{B} = \text{R}174\ 122,48$ Alternatively: $\text{B} = \dfrac{6\ 934,45\left[1-\left(1+\frac{0,185}{12}\right)^{-32}\right]}{\frac{0,185}{12}}$ $\therefore \text{B} = \text{R}174\ 122,75$	✓ growing 250 000 ✓ $n = 28$ ✓ future value formula ✓ $n = 25$ ✓ answer OR ✓✓ present value formula ✓ 6934,45 ✓ $n = 32$ ✓ answer (5)

[13]

SOLUTIONS TO HOMEWORK : SESSION 10
TOPIC: TRANSFORMATIONS

QUESTION 1

1.1.1	$P^{/}(5\,;-2)$	✓ answer (1)
1.1.2	$P^{/}(5\,;2)$	✓ *x*-coordinate ✓ *y*-coordinate (2)
1.2.1	Reduction by a scale factor of $\frac{1}{2}$: $(x\,;y)\rightarrow\left(\frac{1}{2}x\,;\frac{1}{2}y\right)$ Reflection about the line $y=x$ $\left(\frac{1}{2}x\,;\frac{1}{2}y\right)\rightarrow\left(\frac{1}{2}y\,;\frac{1}{2}x\right)$ $\therefore (x\,;y)\rightarrow\left(\frac{1}{2}y\,;\frac{1}{2}x\right)$	✓✓ reduction ✓✓ reflection (4)
1.2.2	If the first transformation is the reflection, then: $H^{/}(8\,;16)$ If the first transformation is the reduction, then: $H^{/}(8\,;4)$	✓ $H^{/}(8\,;16)$ ✓ $H^{/}(8\,;4)$ (2)

1.2.3	$\frac{\text{Area of original}}{\text{Area of image}}=\frac{1}{k^2}$
	$\therefore \frac{\text{Area KUHLE}}{\text{Area K}^{//}\text{U}^{//}\text{H}^{//}\text{L}^{//}\text{E}^{//}}=\frac{1}{\left(\frac{1}{2}\right)^2}=4$
	$\therefore$ Area KUHLE" : Area $\text{K}^{//}\text{U}^{//}\text{H}^{//}\text{L}^{//}\text{E}^{//}=4:1$
		✓✓ answer (2)

[11]

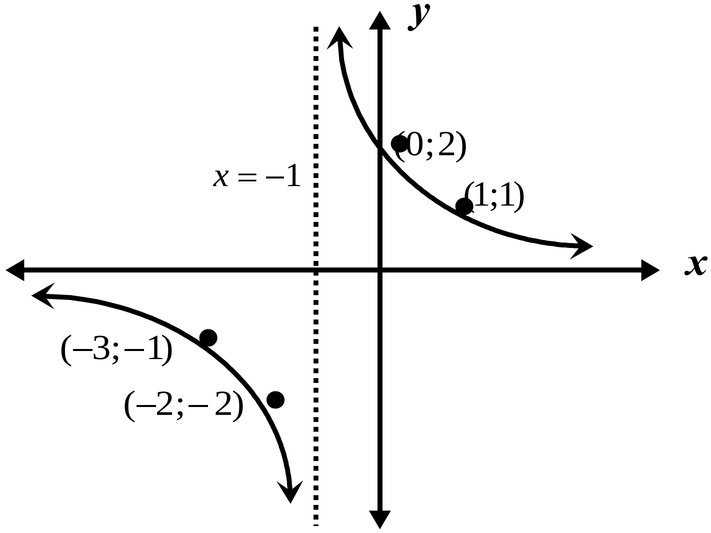
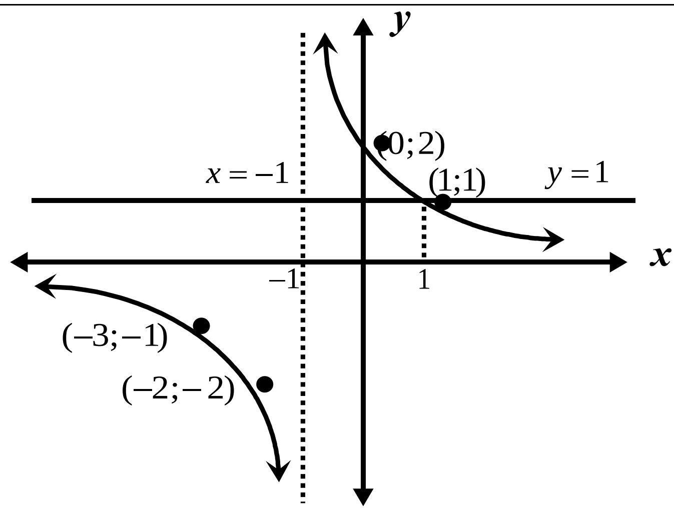
QUESTION 2

2.1	$A^{/}(x\cos\theta - y\sin\theta\ ;\ y\cos\theta + x\sin\theta)$ $x^{/} = x\cos\theta - y\sin\theta$ $\therefore x^{/} = 3\cos120° - 2\sin120°$ $\therefore x^{/} = 3(-\cos60°) - 2\sin60°$ $\therefore x^{/} = -3\left(\frac{1}{2}\right) - 2\left(\frac{\sqrt{3}}{2}\right)$ $\therefore x^{/} = \frac{-3-2\sqrt{3}}{2}$ $y^{/} = y\cos\theta + x\sin\theta$ $\therefore x^{/} = 2\cos120° + 3\sin120°$ $\therefore y^{/} = 2(-\cos60°) + 3\sin60°$ $\therefore y^{/} = -2\left(\frac{1}{2}\right) + 3\left(\frac{\sqrt{3}}{2}\right)$ $\therefore y^{/} = \frac{-2+3\sqrt{3}}{2}$	✓ formula ✓ simplification ✓ substitution ✓ answer ✓ simplification ✓ answer (6)
2.2	$Q(x\ ;\ y) \rightarrow Q^{/}(-2\ ;\ 0)$ $\therefore Q(x\ ;\ y) \rightarrow Q^{/}\left(x\cos120° - y\sin120°\ ;\ y\cos120° + x\sin120°\right)$ $-2 = x\cos120° - y\sin120°$ and $0 = y\cos120° + x\sin120°$ $-2 = x\left(-\frac{1}{2}\right) - y\left(\frac{\sqrt{3}}{2}\right)$ and $0 = y\left(-\frac{1}{2}\right) + x\left(\frac{\sqrt{3}}{2}\right)$ $\therefore -4 = -x - \sqrt{3}y$ and $0 = -y + \sqrt{3}x$ $\therefore 4 = x + \sqrt{3}y$ and $y = \sqrt{3}x$ $\therefore 4 = x + \sqrt{3}(\sqrt{3}x)$ $\therefore 4 = x + 3x$ $\therefore 4 = 4x$ $\therefore x = 1$ $\therefore y = \sqrt{3}(1) = \sqrt{3}$ $\therefore Q(1\ ;\ \sqrt{3})$	✓ $-2 = x\left(-\frac{1}{2}\right) - y\left(\frac{\sqrt{3}}{2}\right)$ ✓ $0 = y\left(-\frac{1}{2}\right) + x\left(\frac{\sqrt{3}}{2}\right)$ ✓ *x*-coordinate ✓ *y*-coordinate (4)

[10]

SOLUTIONS TO HOMEWORK: SESSION 11
TOPIC : FUNCTIONS AND GRAPHS

QUESTION 1

1.1	vertical: $x=-1$ horizontal: $y=0$	✓ vertical ✓ horizontal (2)
1.2		✓ left branch ✓ right branch ✓ coordinates ✓ asymptotes (4)
1.3	$y=\frac{2}{x+1-3}+2$ $\therefore y=\frac{2}{x-2}+2$	✓ $\frac{2}{x+1-3}$ ✓ $y=\frac{2}{x-2}+2$ (2)
1.4	 Therefore $\frac{2}{x+1}\geq 1$ for $-1<x\leq 1$	✓ drawing the line $y=1$ ✓ $(1;1)$ ✓✓ $-1<x\leq 1$ (4)

[12]

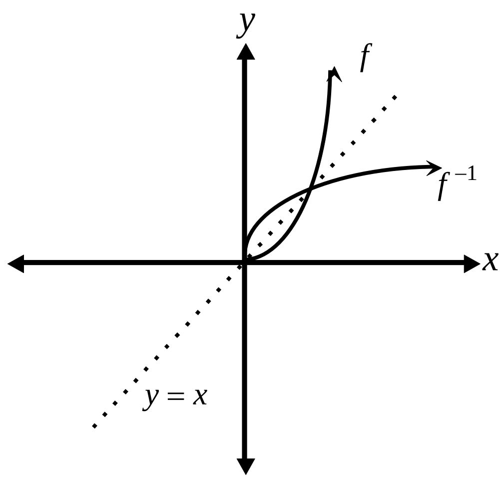
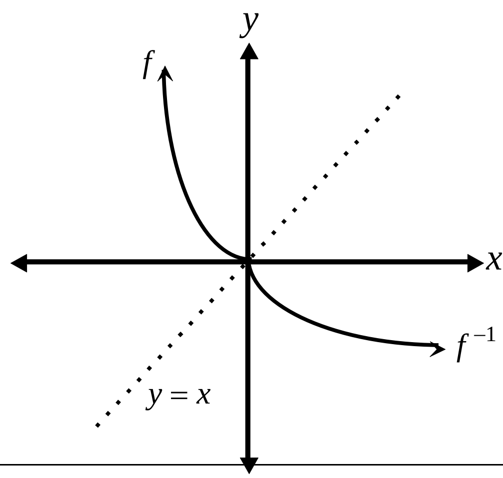
QUESTION 2

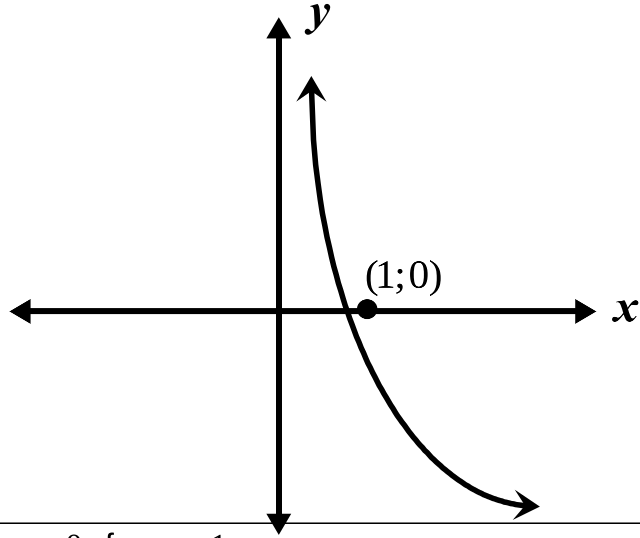
2.1	$y=a(x-1)^2+4$ Substitute $(0\,;3)$ $3=a(0-1)^2+4$ $\therefore 3=a+4$ $\therefore a=-1$ $\therefore f(x)=-(x-1)^2+4$	✓ $y=a(x-1)^2+4$ ✓ $3=a(0-1)^2+4$ ✓ $a=-1$ ✓ $f(x)=-(x-1)^2+4$ (4)
2.2	$y=\frac{a}{x-1}+4$ Substitute $(0\,;3)$ $\therefore 3=\frac{a}{0-1}+4$ $\therefore 3=-a+4$ $\therefore a=1$ $\therefore g(x)=\frac{1}{x-1}+4$	✓✓ $y=\frac{a}{x-1}+4$ ✓ $a=1$ ✓ $g(x)=\frac{1}{x-1}+4$ (4)
2.3	$0=\frac{1}{x-1}+4$ $\therefore 0=1+4(x-1)$ $\therefore 0=1+4x-4$ $\therefore 0=4x-3$ $\therefore -4x=-3$ $\therefore x=\frac{3}{4}$ $\left(\frac{3}{4}\,;0\right)$	✓ $0=\frac{1}{x-1}+4$ ✓ $x=\frac{3}{4}$ ✓ $\left(\frac{3}{4}\,;0\right)$ (3)
2.4	$g(x)\le 0$ for $0\le x<\frac{3}{4}$	✓ $x\ge 0$ ✓ $x<\frac{3}{4}$ (2)

[13]

SOLUTIONS TO HOMEWORK: SESSION 12
TOPIC: INVERSE GRAPHS

QUESTION 1

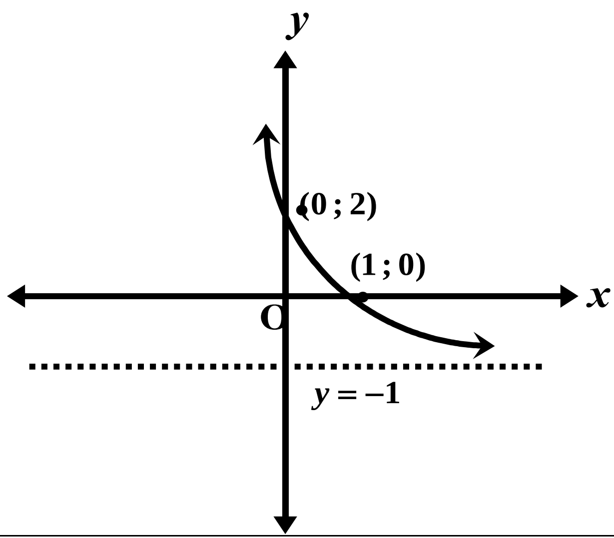
1.1	$f(x)=2x^2$ where $x\geq 0$ OR $f(x)=2x^2$ where $x\leq 0$	✓ $x\geq 0$ OR $x\leq 0$ (1)
1.2	 OR  	✓ f ✓ f^{-1} (2)
1.3	$y=\left(\frac{1}{2}\right)^x$ $x=\left(\frac{1}{2}\right)^y$ $\therefore \log_{\frac{1}{2}} x = y$ $\therefore g^{-1}(x)=\log_{\frac{1}{2}} x$	✓ $x=\left(\frac{1}{2}\right)^y$ ✓ $g^{-1}(x)=\log_{\frac{1}{2}} x$ (2)

1.4		✓ shape ✓ $(1;0)$ (2)
1.5	$\log_{\frac{1}{2}} x < 0$ for $x > 1$	✓ $x > 1$ (1)

[8]

QUESTION 2

2.1	$y = 2a^x$ Substitute $(-1;4)$ $\therefore 4 = 2a^{-1}$ $\therefore 4 = \frac{2}{a}$ $\therefore 4a = 2$ $\therefore a = \frac{1}{2}$ $\therefore f(x) = 2\left(\frac{1}{2}\right)^x$	✓ $4 = 2a^{-1}$ ✓ $a = \frac{1}{2}$ ✓ $f(x) = 2\left(\frac{1}{2}\right)^x$ (3)
2.2	$y = 2\left(\frac{1}{2}\right)^0 = 2$ $(0;2)$	✓ $2\left(\frac{1}{2}\right)^0$ ✓ $(0;2)$ (2)

2.3	$g(x)=2\left(\frac{1}{2}\right)^{-x}$ $\therefore g(x)=2(2^{x})$ $\therefore g(x)=2^{1+x}$ $\therefore g(x)=2^{x+1}$	✓ $g(x)=2\left(\frac{1}{2}\right)^{-x}$ ✓ $g(x)=2(2^{x})$ ✓ $g(x)=2^{x+1}$ (3)
2.4	 	✓ *y*-intercept ✓ *x*-intercept ✓ asymptote ✓ shape (4) **[12]**

SOLUTIONS TO HOMEWORK: SESSION 13

TOPIC 1: CALCULUS – LIMITS AND FIRST PRINCIPLES

1.1.1	$f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $\therefore f'(x)=\lim_{h\to 0}\dfrac{1-\frac{1}{4}(x+h)^2-\left(1-\frac{1}{4}x^2\right)}{h}$ $\therefore f'(x)=\lim_{h\to 0}\dfrac{1-\frac{1}{4}(x^2+2xh+h^2)-1+\frac{1}{4}x^2}{h}$ $\therefore f'(x)=\lim_{h\to 0}\dfrac{1-\frac{1}{4}x^2-\frac{1}{2}xh-\frac{1}{4}h^2-1+\frac{1}{4}x^2}{h}$ $\therefore f'(x)=\lim_{h\to 0}\dfrac{-\frac{1}{2}xh-\frac{1}{4}h^2}{h}$ $\therefore f'(x)=\lim_{h\to 0}\dfrac{h\left(-\frac{1}{2}x-\frac{1}{4}h\right)}{h}$ $\therefore f'(x)=\lim_{h\to 0}\left(-\frac{1}{2}x-\frac{1}{4}h\right)$ $\therefore f'(x)=-\frac{1}{2}x$	✓ $1-\frac{1}{4}(x+h)^2$ ✓ $-1+\frac{1}{4}x^2$ ✓ $-\frac{1}{4}x^2-\frac{1}{2}xh-\frac{1}{4}h^2$ ✓ $-\frac{1}{2}xh-\frac{1}{4}h^2$ ✓ $\left(-\frac{1}{2}x-\frac{1}{4}h\right)$ ✓ $-\frac{1}{2}x$ (6)
1.1.2	$f'(-4)=-\frac{1}{2}(-4)=2$	✓ answer (1)
1.1.3	$f(x)=1-\frac{1}{4}x^2$ $\therefore f(-2)=1-\frac{1}{4}(-2)^2$ $\therefore f(-2)=0$ The answer represents the *y*-value corresponding to $x=-2$	✓ $f(-2)=0$ ✓ interpretation (2)

1.1.4	$f(x)=1-\frac{1}{4}x^2$ $f(-2)=0$ $f(4)=1-\frac{1}{4}(4)^2$ $\therefore f(4)=-3$ $(-2\,;0)$ and $(4\,;-3)$ Average gradient $=\frac{-3-0}{4-(-2)}=\frac{-3}{6}=-\frac{1}{2}$	✓ $f(-2)=-7$ ✓ $f(4)=-31$ ✓ $\frac{-31-(-7)}{4-(-2)}$ ✓ -4 (4)
1.2	$\therefore f'(x)=\lim_{h\to 0}\frac{-\frac{3}{x+h}-\left(-\frac{3}{x}\right)}{h}$ $\therefore f'(x)=\lim_{h\to 0}\frac{-\frac{3}{x+h}+\frac{3}{x}}{h}$ $\therefore f'(x)=\lim_{h\to 0}\frac{\frac{-3x+3(x+h)}{x(x+h)}}{h}$ $\therefore f'(x)=\lim_{h\to 0}\frac{\frac{-3x+3x+3h}{x(x+h)}}{h}$ $\therefore f'(x)=\lim_{h\to 0}\frac{\frac{3h}{x(x+h)}}{h}$ $\therefore f'(x)=\lim_{h\to 0}\frac{3h}{x(x+h)}\times\frac{1}{h}$ $\therefore f'(x)=\lim_{h\to 0}\frac{3}{x(x+h)}$ $\therefore f'(x)=\frac{3}{x(x+0)}$ $\therefore f'(x)=\frac{3}{x^2}$	✓ $-\frac{3}{x+h}-\left(-\frac{3}{x}\right)$ ✓ $-\frac{3}{x+h}+\frac{3}{x}$ ✓ $\frac{\frac{3h}{x(x+h)}}{h}$ ✓ $\frac{3}{x(x+h)}$ ✓ $\frac{3}{x^2}$ (5)

[18]

QUESTION 2

2.1	$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{-2(x+h)-(-2x)}{h}$ $=\lim_{h\to 0}\frac{-2x-2h+2x}{h}$ $=\lim_{h\to 0}\frac{-2h}{h}$ $=\lim_{h\to 0}(-2)$ $=-2$	✓ $-2(x+h)-(-2x)$ ✓ $-x^2+2x$ ✓ $\frac{-2h}{h}$ ✓ -2 (4)
2.2	$g'(x)=\lim_{h\to 0}\frac{-2(x+h)^3-(-2x^3)}{h}$ $\therefore g'(x)=\lim_{h\to 0}\frac{-2(x+h)(x+h)^2+2x^3}{h}$ $\therefore g'(x)=\lim_{h\to 0}\frac{-2(x+h)(x^2+2xh+h^2)+2x^3}{h}$ $\therefore g'(x)=\lim_{h\to 0}\frac{-2(x^3+2x^2h+xh^2+x^2h+2xh^2+h^3)+2x^3}{h}$ $\therefore g'(x)=\lim_{h\to 0}\frac{-2(x^3+3x^2h+3xh^2+h^3)+2x^3}{h}$ $\therefore g'(x)=\lim_{h\to 0}\frac{-2x^3-6x^2h-6xh^2-2h^3+2x^3}{h}$ $\therefore g'(x)=\lim_{h\to 0}\frac{-6x^2h-6xh^2-2h^3}{h}$ $\therefore g'(x)=\lim_{h\to 0}\frac{h(-6x^2-6xh-2h)}{h}$ $\therefore g'(x)=\lim_{h\to 0}(-6x^2-6xh-2h)$ $\therefore g'(x)=-6x^2$ $\therefore g'(2)=-6(2)^2=-24$	✓ $-2(x+h)^3-(-2x^3)$ ✓✓ $-6x^2h-6xh^2-2h^3$ ✓✓ $(-6x^2-6xh-2h)$ ✓ $-6x^2$ (6)

[10]

SOLUTIONS TO HOMEWORK: SESSION 13

TOPIC 2: CALCULUS – RULES OF DIFFERENTIATION AND TANGENTS

QUESTION 1

1.1	$f(x)=(4x-3)^2$ $\therefore f(x)=16x^2-24x+9$ $\therefore f'(x)=16\times 2x^{2-1}-24+0$ $\therefore f'(x)=32x-24$	✓ $16x^2-24x+9$ ✓ $32x-24$ (2)
1.2	$D_x\left[\sqrt[3]{x}+\frac{1}{\sqrt{x}}\right]$ $=D_x\left[x^{\frac{1}{3}}+x^{-\frac{1}{2}}\right]$ $=\frac{1}{3}x^{-\frac{2}{3}}-\frac{1}{2}x^{-\frac{3}{2}}$ $=\frac{1}{3x^{\frac{2}{3}}}-\frac{1}{2x^{\frac{3}{2}}}$	✓ $x^{\frac{1}{3}}+x^{-\frac{1}{2}}$ ✓ $\frac{1}{3}x^{-\frac{2}{3}}-\frac{1}{2}x^{-\frac{3}{2}}$ ✓ $\frac{1}{3x^{\frac{2}{3}}}-\frac{1}{2x^{\frac{3}{2}}}$ (3)
1.3	$D_x\left[(x^2-\sqrt{x})^2\right]$ $=D_x\left[x^4-2x^2\sqrt{x}+x\right]$ $=D_x\left[x^4-2x^2x^{\frac{1}{2}}+x\right]$ $=D_x\left[x^4-2x^{\frac{5}{2}}+x\right]$ $=4x^3-5x^{\frac{3}{2}}+1$	✓✓ $x^4-2x^{\frac{5}{2}}+x$ ✓✓✓ $4x^3-5x^{\frac{3}{2}}+1$ (5)

1.4	$y=\frac{2x^2-\sqrt{x}+5}{\sqrt{x}}$ $\therefore y=\frac{2x^2}{x^{\frac{1}{2}}}-1+\frac{5}{x^{\frac{1}{2}}}$ $\therefore y=2x^{\frac{3}{2}}-1+5x^{-\frac{1}{2}}$ $\therefore \frac{dy}{dx}=3x^{\frac{1}{2}}-\frac{5}{2}x^{-\frac{3}{2}}$ $\therefore \frac{dy}{dx}=3x^{\frac{1}{2}}-\frac{5}{2x^{\frac{3}{2}}}$	✓ $x^{\frac{1}{2}}$ ✓ $2x^{\frac{3}{2}}-1+5x^{-\frac{1}{2}}$ ✓ $3x^{\frac{1}{2}}-\frac{5}{2}x^{-\frac{3}{2}}$ ✓ $3x^{\frac{1}{2}}-\frac{5}{2x^{\frac{3}{2}}}$ (4)

[14]

QUESTION 2

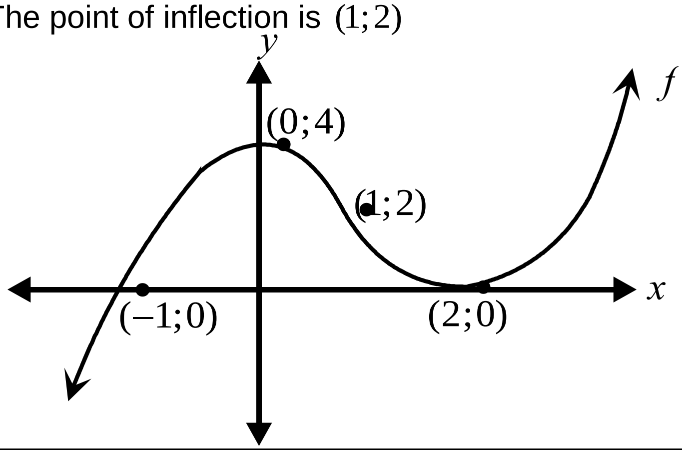
2.1	$y=f(x)=3x^2-2x+2$ $x_{\mathrm{T}}=-4$ $\therefore f(-4)=3(-4)^2-2(-4)+2$ $\therefore f(-4)=58$ $\therefore y_{\mathrm{T}}=58$ $m_{\mathrm{T}}=f'(x)=6x-2$ $\therefore f'(-4)=6(-4)-2$ $\therefore f'(5)=-26$ $y-y_{\mathrm{T}}=m_t(x-x_{\mathrm{T}})$ $\therefore y-58=-26(x-(-4))$ $\therefore y-58=-26x-104$ $\therefore y=-26x-46$	✓ $f(-4)=58$ ✓ $f'(x)=6x-2$ ✓ $f'(5)=-26$ ✓ $y-58=-26(x-(-4))$ ✓ $y=-26x-46$ (5)

2.2	$y = a(x-0)(x-4)$ Substitute the point $(3\,;6)$ $\therefore 6 = a(3-0)(3-4)$ $\therefore 6 = -3a$ $\therefore a = -2$ $\therefore y = -2(x-0)(x-4)$ $\therefore y = -2x(x-4)$ $\therefore y = -2x^2 + 8x$ $f(x) = -2x^2 + 8x$ $\therefore f'(x) = -4x + 8$ $\therefore f'(3) = -4(3) + 8 = -4$ $y - y_T = m_t(x - x_T)$ $\therefore y - 6 = -4(x-3)$ $\therefore y - 6 = -4x + 12$ $\therefore y = -4x + 18$	✓ $y = a(x-0)(x-4)$ ✓ $a = -2$ ✓ $f(x) = -2x^2 + 8x$ ✓ $f'(x) = -4x + 8$ ✓ $f'(3) = -4(3) + 8 = -4$ ✓ $y = -4x + 18$ (6)

[11]

SOLUTIONS TO HOMEWORK: SESSION 14
TOPIC 1: CALCULUS – GRAPHICAL APPLICATIONS

QUESTION 1

	x-intercepts: *y*-intercept: 4 $0 = x^3 - 3x^2 + 4$ $\therefore (x+1)(x^2 - 4x + 4) = 0$ $\therefore (x+1)(x-2)(x-2) = 0$ $\therefore x = -1$ or $x = 2$	✓ $0 = x^3 - 3x^2 + 4$ ✓ $(x+1)(x^2 - 4x + 4) = 0$ ✓ $x = -1$ or $x = 2$
	$f'(x) = 3x^2 - 6x$ $\therefore 0 = 3x^2 - 6x$ $\therefore 0 = x^2 - 2x$ $\therefore 0 = x(x-2)$ $\therefore x = 0$ or $x = 2$	✓ $f'(x) = 3x^2 - 6x$ ✓ $0 = 3x^2 - 6x$ ✓ $x = 0$ or $x = 2$
	For $x = 0$ $f(0) = (0)^3 - 3(0)^2 + 4 = 4$ Max turning point at $(0\,;4)$ For $x = 2$ $f(2) = (2)^3 - 3(2)^2 + 4 = 0$ Min turning point at $(2\,;0)$	✓ $(0\,;4)$ ✓ $(2\,;0)$
	$f'(x) = 3x^2 - 6x$ $\therefore f''(x) = 6x - 6$ $\therefore 0 = 6x - 6$ $\therefore -6x = -6$ $\therefore x = 1$ $f(1) = (1)^3 - 3(1)^2 + 4$ $f(1) = 2$ The point of inflection is $(1;2)$	✓ $f''(x) = 6x - 6$ ✓ $x = 1$ ✓ $(1;2)$
		✓ intercepts with the axes ✓ turning points ✓ shape ✓ point of inflection **[15]**

QUESTION 2

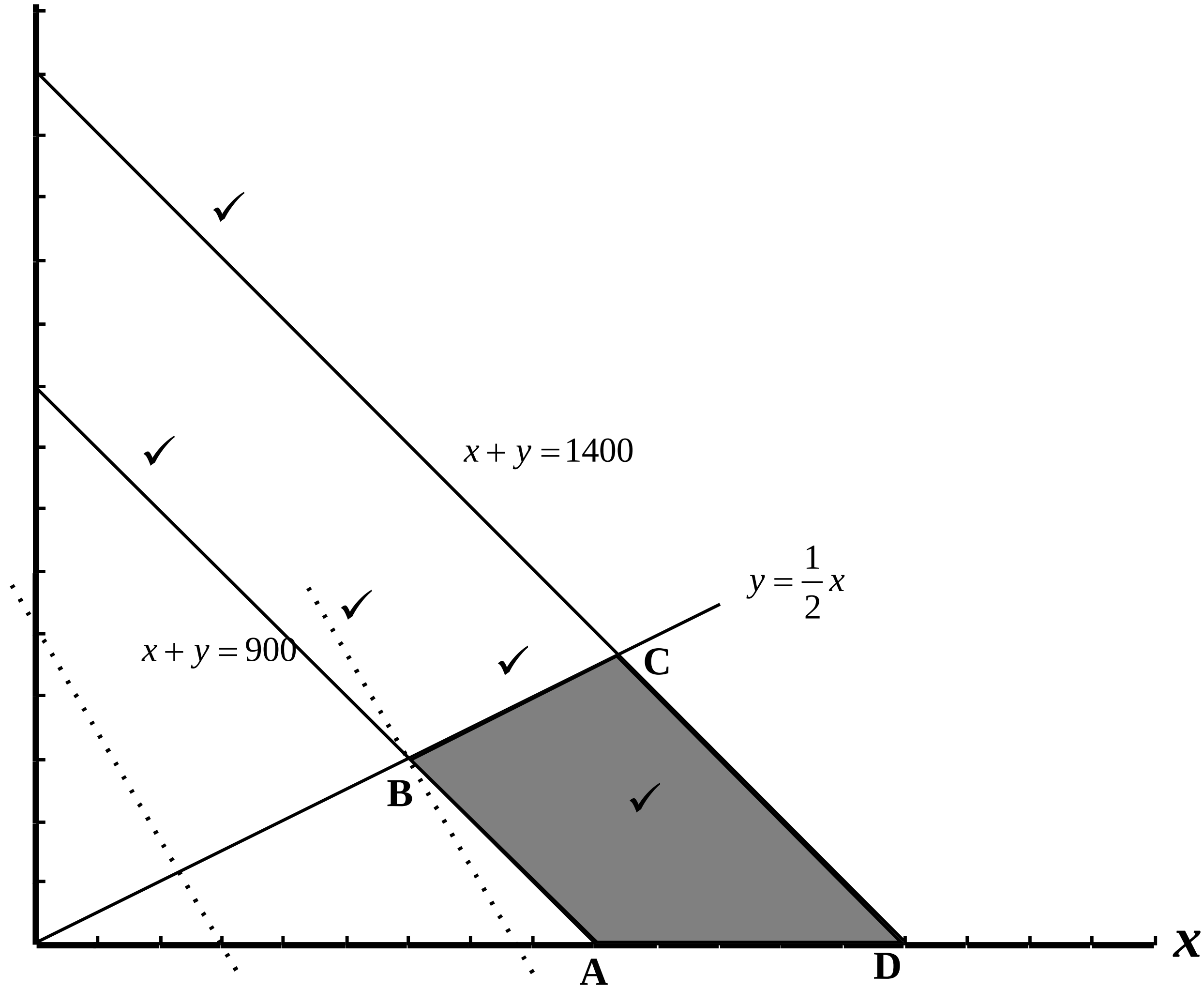
2.1	$h'(x) = -3x^2 + 2ax + b$ $h'(-1) = -3(-1)^2 + 2a(-1) + b$ $0 = -3 - 2a + b$ $2a - b = -3 \quad \ldots \text{(i)}$ $h'(2) = -3(2)^2 + 2a(2) + b$ $0 = -12 + 4a + b$ $4a + b = 12 \quad \ldots \text{(ii)}$ $6a = 9 \qquad \text{(i)} + \text{(ii)}$ $\therefore a = \frac{3}{2}$ $\therefore 2\left(\frac{3}{2}\right) - b = -3$ $b = 6$	✓ $h'(x) = -3x^2 + 2ax + b$ ✓ $h'(-1) = -3(-1)^2 + 2a(-1) + b$ ✓ $2a - b = -3$ ✓ $h'(2) = -3(2)^2 + 2a(2) + b$ ✓ $4a + b = 12$ ✓ $a = \frac{3}{2}$ ✓ $b = 6$ (7)
2.2	Average gradient $= \frac{10 - (-3,5)}{2 - (-1)}$ $= \frac{13,5}{3}$ $= \frac{9}{2}$	✓ $\frac{10 - (-3,5)}{2 - (-1)}$ ✓ $\frac{9}{2}$ (2)
2.3	$h(x) = -x^3 + \frac{3}{2}x^2 + 6x$ $\therefore h'(x) = -3x^2 + 3x + 6$ $h'(-2) = -3(-2)^2 + 3(-2) + 6$ $h'(-2) = -12$ Point of contact $(-2\ ;\ 2)$ $y - 2 = -12(x + 2)$ $y = -12x - 22$	✓ $h(x) = -x^3 + \frac{3}{2}x^2 + 6x$ ✓ $h'(x) = -3x^2 + 3x + 6$ ✓ $h'(-2) = -12$ ✓ $y = -12x - 22$ ✓ $h'(-2) = -12$ (5)
2.4	$h'(x) = -3x^2 + 3x + 6$ $h''(x) = -6x + 3$ $-6x + 3 = 0$ $x = \frac{1}{2}$	✓ $h''(x) = -6x + 3$ ✓ $-6x + 3 = 0$ ✓ $x = \frac{1}{2}$ (3)

[17]

SOLUTIONS TO HOMEWORK: SESSION 14
TOPIC 2: LINEAR PROGRAMMING

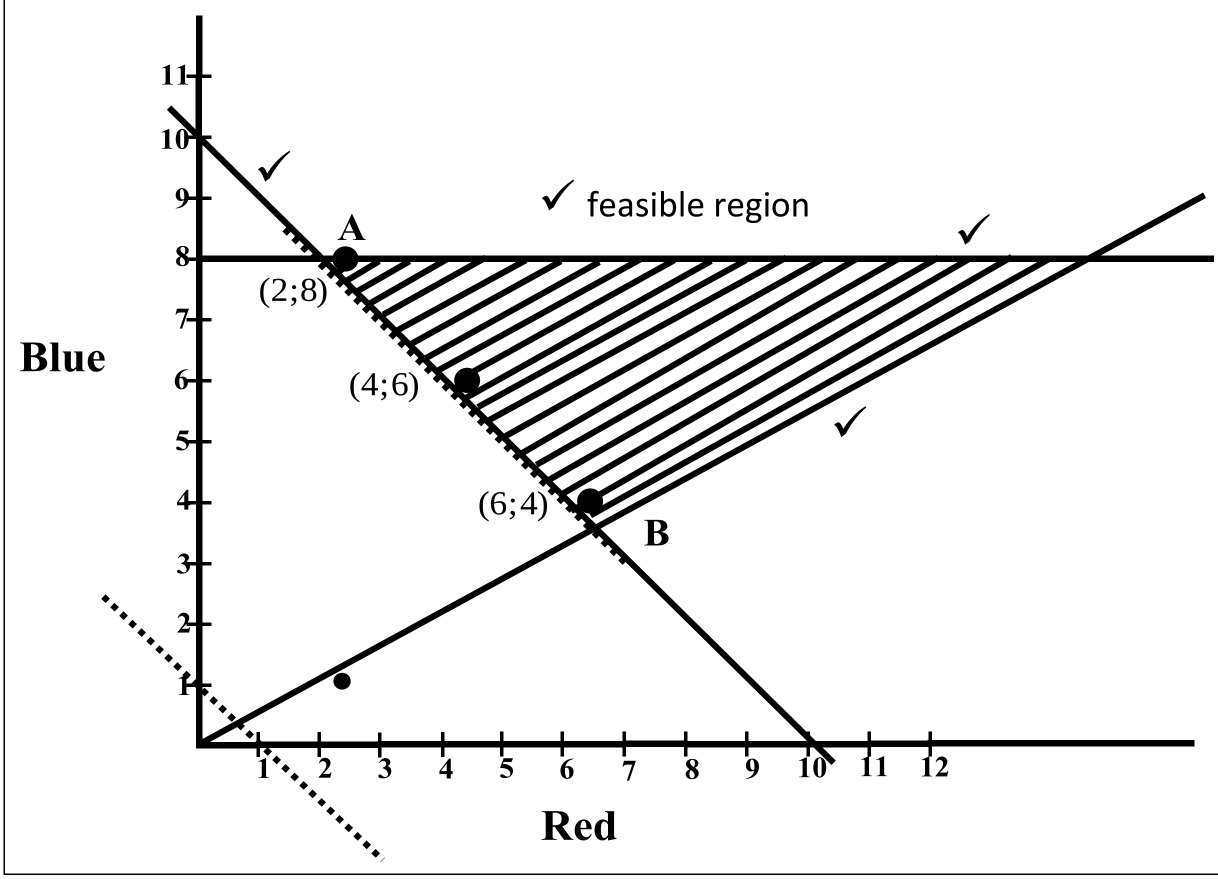
QUESTION 1

Let $x =$ chemical P and $y =$ chemical Q $x+y\geq 900$ $\quad x+y\leq 1400$ $\quad y\leq \frac{1}{2}x$ The objective function is $C=5x+3y$ $\therefore 5x+3y=C$ $\therefore 3y=-5x+C$ $\therefore y=-\frac{5}{3}x+\frac{C}{3}$ Cut on *y*-axis is 5 Cut on *x*-axis is 3 The most cost effective mixture is the point at which the cost is a minimum.The minimum cost is at point B$(600\,;300)$. Therefore, the most cost effective mixture is: 600kg of P and 300kg of Q.	✓ $x+y\geq 900$ ✓ $x+y\leq 1400$ ✓✓ $y\leq \frac{1}{2}x$ ✓ $C=5x+3y$ ✓ $y=-\frac{5}{3}x+\frac{C}{3}$ ✓ 600kg of P ✓ 300kg of Q See graph for other marks **[13]**



QUESTION 2

2.1	$x+y\geq 10$ $y\geq \frac{1}{2}x$ $y\leq 8$	✓ $x+y\geq 10$ ✓ $y\geq \frac{1}{2}x$ ✓ $y\leq 8$ (3)
2.2	see below	
2.3	$C=40x+40y$ $\therefore 40x+40y=C$ $\therefore 40y=-40x+C$ $\therefore y=-1x+\frac{C}{40}$	✓ $C=40x+40y$ ✓ search line on diagram ✓ $(2;8)$ ✓ $(4;6)$ ✓ $(6;4)$ (5)



(4)
[12]